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VOLATILITY MODELLING USING GARCH FAMILY MODELS: A COMPARISON OF NIFTY ESG 100 INDEX AND MSCI EUROPE ESG LEADERS INDEX

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Abstract

This study explores the volatility dynamics of the Nifty ESG 100 Index using the ARCH family of models**.** The Autoregressive Conditional Heteroscedasticity (ARCH) model and its generalizations, particularly the Generalized ARCH (GARCH) model, have proven to be effective in capturing time-varying volatility in financial time series. The Nifty ESG 100 Index, which tracks companies with high environmental, social, and governance (ESG) standards, is becoming increasingly relevant in India's evolving financial markets. By employing ARCH, GARCH, and extensions like EGARCH and TGARCH, this paper seeks to model and forecast the index's volatility, considering the persistence and asymmetric behavior of market fluctuations. The analysis aims to provide insights into the volatility patterns that can assist investors and portfolio managers in making informed risk management decisions. Diagnostic tests confirm the validity of the models, and out-of-sample volatility forecasts highlight the robustness of GARCH-type models in capturing volatility clustering and persistence for the Nifty ESG 100 Index.

Keywords: GARCH Models; ESG Markets; Developing and Developed Countries

1. Introduction

Volatility modelling plays a critical role in financial econometrics, particularly for understanding and forecasting risk in asset prices. In recent years, sustainable investing has gained traction, with indices like the Nifty ESG 100 Index being introduced to track the performance of companies that meet environmental, social, and governance (ESG) criteria. These indices represent a new frontier in investment strategies, blending financial performance with sustainability objectives. Given the growing importance of ESG-focused indices, understanding their volatility is essential for investors looking to align financial goals with ethical considerations.

The Nifty ESG 100 Index, launched by the National Stock Exchange (NSE) in India, includes the top 100 companies based on ESG scores from the Nifty 100 universe. The index has attracted attention due to the increasing awareness of sustainable practices, but like other financial indices, it is subject to market volatility driven by both local and global factors. Understanding this volatility is critical for portfolio management, hedging strategies, and risk assessment.

In financial markets, volatility often exhibits time-varying behaviour and is prone to clustering, where periods of high volatility are followed by more volatile periods, and periods of calmness follow periods of low volatility. Such characteristics make ARCH models, proposed by Engle (1982), particularly useful in volatility analysis. The basic ARCH model was later generalized by Bollerslev (1986) to the GARCH model, allowing for both past squared returns and past volatility to explain current volatility. These models have been widely applied to a variety of asset classes, including stocks, bonds, and commodity markets, to capture their time-varying volatility.

This paper applies the ARCH family of models to the Nifty ESG 100 Index, aiming to model and forecast its volatility. By using GARCH (1, 1) and more advanced models such as EGARCH and TGARCH, we account for volatility clustering, persistence, and asymmetric behaviour in the data. These models are particularly valuable in capturing the leverage effect, where negative shocks tend to increase volatility more than positive shocks, a feature often observed in equity markets.

The primary contributions of this paper are twofold: first, to provide a detailed analysis of the volatility structure of a relatively new and important index in India, and second, to offer out-of-sample forecasts of volatility that can guide investment decisions and risk management strategies. Through the use of diagnostic tests and model comparisons, we validate the effectiveness of these models in explaining the volatility patterns of the Nifty ESG 100 Index.

2. Review of Literature

Volatility modeling has been a central focus in financial econometrics, particularly since the introduction of autoregressive conditional heteroskedasticity (ARCH) by Engle (1982) and its generalized version (GARCH) by Bollerslev (1986). These models have proven indispensable for understanding the time-varying nature of financial volatility. Over time, numerous variants of the GARCH model, such as TGARCH, EGARCH, PARCH, and IGARCH, have been developed to account for asymmetries, leverage effects, and persistence in volatility, further improving risk management and portfolio strategies across different asset classes and markets.

2.1 Volatility Modeling with GARCH Family Models

The traditional GARCH (1,1) model proposed by Bollerslev (1986) remains one of the most widely used models to capture volatility clustering, where periods of high volatility are followed by high volatility, and periods of low volatility are followed by low volatility. It assumes that volatility is time-varying but symmetrically distributed, which can limit its accuracy in the presence of market shocks. Nelson (1991) introduced the Exponential GARCH (EGARCH) model, which allows for the modeling of asymmetric volatility, capturing the "leverage effect," where negative shocks tend to increase volatility more than positive shocks of the same magnitude.

In addition to EGARCH, Glosten, Jagannathan, and Runkle (1993) proposed the Threshold GARCH (TGARCH) model to accommodate the asymmetric response of volatility to negative news. These models improve the accuracy of volatility forecasting, especially during periods of financial distress, making them valuable tools for analyzing market behavior in both developed and emerging markets. PARCH (Power ARCH), introduced by Ding, Granger, and Engle (1993), further refines volatility modeling by allowing for power transformations of the absolute value of returns, making the model more flexible in capturing volatility patterns across various asset classes.

Empirical research has demonstrated the efficacy of these models in capturing volatility dynamics across different markets. For instance, studies like Franses and Van Dijk (1996) have emphasized the advantages of GARCH models in emerging markets, where volatility tends to be higher and more persistent. More recently, volatility modeling has gained attention in sustainable finance, particularly in the context of ESG (Environmental, Social, and Governance) indices.

2.2 Volatility in ESG Markets

The rise of ESG investing has prompted research into the volatility characteristics of ESG indices. Research by Reboredo et al. (2017) examines the volatility of green bond indices and finds that while they exhibit lower volatility compared to traditional indices, they are still sensitive to market shocks. Studies like Ardia et al. (2021) focus on ESG equity indices and note that while ESG stocks generally display lower volatility due to their more resilient business models and governance structures, they are not immune to periods of heightened volatility caused by global financial and economic events. The NIFTY ESG 100 Index, which tracks companies in India that score well on environmental, social, and governance criteria, has become an area of focus for volatility modeling in emerging markets.

In the European context, indices like the MSCI Europe ESG Leaders Index have been examined for their volatility characteristics. Studies such as Menike (2022) note that ESG indices in developed markets tend to exhibit lower volatility compared to traditional indices, although they can still be affected by macroeconomic factors, geopolitical risks, and environmental crises. The ESG factors' ability to absorb market shocks and potentially reduce downside risk has led to the increasing use of GARCH models to compare volatility dynamics between ESG and non-ESG indices.

2.3 Comparative Studies of Volatility in Emerging vs. Developed Markets

Emerging markets, such as India, are typically characterized by higher volatility due to macroeconomic instability, regulatory changes, and liquidity issues (Bekaert and Harvey, 1997). In contrast, developed markets, such as Europe, often exhibit lower volatility but are still subject to market contagion and spillover effects. When comparing volatility in emerging markets with developed markets, studies like Baele (2005) highlight the importance of modeling volatility spillovers. Emerging market indices such as the NIFTY ESG 100 may experience greater volatility clustering due to local factors, while indices like MSCI Europe ESG Leaders may experience volatility spillovers from global economic events.

The use of GARCH-type models for analyzing spillover effects between markets has become increasingly common. Engle and Rangel (2008) introduced the concept of time-varying correlation through a Dynamic Conditional Correlation (DCC) GARCH model, which has been employed in cross-market studies to capture spillover effects. Bouri et al. (2020) use GARCH models to study the impact of global market movements on ESG indices, revealing that volatility spillovers are significant in both directions, although more pronounced from developed to emerging markets.

2.4 Leverage Effect and Asymmetric Volatility

One of the key findings in volatility modeling using GARCH family models is the leverage effect, where negative returns increase volatility more than positive returns of the same magnitude (Black, 1976). The asymmetric nature of volatility, as captured by models like EGARCH and TGARCH, is particularly relevant when studying indices such as the NIFTY ESG 100 and MSCI Europe ESG Leaders. Research by Chen and Ghysels (2011) confirms the presence of significant asymmetry in emerging markets, while Brooks and Persand (2003) show that European markets also exhibit leverage effects, though to a lesser extent. This asymmetric behavior underscores the importance of using models that can capture non-linear responses to market events.

Volatility modeling using GARCH family models has proven to be a valuable tool in understanding the risk dynamics of both traditional and ESG indices. As ESG investing continues to grow, understanding the volatility characteristics of ESG indices becomes increasingly important. The comparison between the NIFTY ESG 100 Index and the MSCI Europe ESG Leaders Index provides insights into how emerging and developed markets behave under different volatility regimes. The use of GARCH, TGARCH, EGARCH, and other related models allows for a deeper understanding of how these markets respond to shocks, including the leverage and spillover effects. Future research could expand on these findings by incorporating macroeconomic variables and employing multivariate GARCH models to further analyze the co-movement between ESG and traditional indices.

3. Data and Research Methodology

Select Indices for the Study

India's ESG Index: Nifty ESG 100

India's Benchmark Index: Nifty 50

EU's ESG Index: MSCI Europe ESG Leaders

EU's Benchmark Index: EURO STOXX 50

The analysis covers a 7-year period (2017-2024) and focuses on the following GARCH family results.

3.1 The GARCH Model

GARCH models well-defined as models of returns or of financial time series. Here, volatility corresponds with the magnitude of returns of series or of some other financial time series. This is because of assuming the series to be modelled is proportional to a zero-mean. Volatility depends not only on the past prices of the process as in ARCH models but also on the past prices of volatility. The simplest GARCH (1, 1) specification as follows: in which the mean equation given in (1) is written as a function of exogenous variables with an error term.

The GARCH (1, 1) model is represented by the following two key equations:

The Mean Equation is $$Y_t = X_t{}' \theta + \epsilon_t \quad (1)$$

The Variance Equation is $$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (2)$$

In this model, the conditional variance σ_t^2 depends on both the past shocks ϵ_{t-1}^2 and the previous period's variance $\beta\sigma_{t-1}^2$.

3.2 TGARCH Model

The Threshold GARCH (TGARCH) model is an extension of the GARCH model that accounts for the asymmetric effects of positive and negative shocks on volatility. It allows the conditional variance to respond differently to positive and negative innovations, often capturing the phenomenon known as the "leverage effect," where negative shocks tend to increase volatility more than positive shocks of the same magnitude.

The TGARCH (1,1) model is typically expressed as:

$$\sigma_t^2 = \omega + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-1}^2 + \sum_{k=1}^{r} \gamma_k \epsilon_t^2 \Gamma_{t-k} \quad (3)$$

In this model, if $\gamma_1 > 0$, negative shocks (bad news) increase volatility more than positive shocks (good news) of the same magnitude. TGARCH is useful in capturing these asymmetries, especially in financial time series where bad news tends to have a larger impact on volatility than good news.

3.3 EGARCH Model

The exponential GARCH model was proposed by Nelson (1991). This model a variant of the GARCH model that captures asymmetry and leverage effects, allowing volatility to respond differently to positive and negative shocks. Unlike the standard GARCH, the EGARCH model ensures that the conditional variance is always positive without needing parameter restrictions, as it models the logarithm of the conditional variance. There are various ways to express the conditional variance equation, but one possible specification is given by

$$\log \sigma_t^2 = \omega + \sum_{j=1}^{q} \beta_j \log \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \left| \frac{\epsilon_{t-i}}{\sigma_{t-i}} \right| + \sum_{k=1}^{r} \gamma_k \frac{\epsilon_{t-k}}{\sigma_{t-k}} \quad (4)$$

The model accounts for the asymmetry through the α term: if α≠0, it indicates that positive and negative shocks have different impacts on volatility. If α<0, negative shocks (bad news) increase volatility more than positive shocks (good news).

Because the variance is modeled in logarithmic form, the EGARCH model ensures non-negative variance without imposing constraints on the parameters, making it more flexible for financial time series data that exhibit leverage effects.

3.4 PARCH Model

The **PARCH model** (Power ARCH) is a variant of the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model. It introduces a flexible approach to model the volatility of time series by incorporating a power transformation of the error terms in the variance equation. Unlike traditional GARCH models, where the volatility is modeled using squared residuals, PARCH allows the power of residuals to be any real number, offering more flexibility in capturing different types of volatility patterns.

$$\sigma_t^{\delta} = \omega + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^{\delta} + \sum_{i=1}^{\rho} \alpha_i \left(|\epsilon_{t-i}| - \gamma_i \epsilon_{t-i}\right)\delta \quad (5)$$

Where δ > 1, $|\gamma_i| \leq 1$ for $i = 1, \ldots, r$, $\gamma_i = 0$ for all $i > \mathfrak{r}$, and $\mathfrak{r} \leq p$. The symmetric model sets $\gamma_i$for all i. Note that if $\delta = 2$ and $\gamma_i = 0$ for all i, the PARCH model is simply a standard GARCH specification. As in the previous models, the asymmetric effects are present if $\gamma \neq 0$.

3.5 IGARCH Model

The Integrated GARCH (IGARCH) model is a specialized variant of the GARCH model that is used to model financial time series with persistent volatility clustering. Unlike standard GARCH models, which allow for mean-reverting behavior in the volatility process, IGARCH models incorporate a unit root in the volatility equation, resulting in non-stationary volatility that exhibits long memory properties. This characteristic makes IGARCH particularly suitable for modeling financial data that display high persistence in volatility over time.

$$\sigma_t^2 = \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 \quad (6)$$

Such that

$$\sum_{j=1}^{q} \beta_j + \sum_{i=1}^{p} \alpha_i = 1 \quad (7)$$

then we have an integrated GARCH (Engle and Bollerslev (1986)).

3.6 Hypotheses for the Study

3.6.1 Measures of Normality

Null Hypothesis (H_{01}) : Indices are normally distributed

Alternative Hypothesis (H_{11}) : Indices are not normally distributed

3.6.2 Checking Stationarity

Null Hypothesis (H_{02}) : Indices are not stationary data and has unit root

Alternative Hypothesis (H_{22}) : Indices are stationary data and has no unit root

3.6.3 Presence of Arch Effect

Null Hypothesis (H_{03}) : Presence of Arch effect is not exist

Alternative Hypothesis (H_{33}) : Presence of Arch effect exist

Augmented Dickey – Fuller (ADF) Unit Root Test

The ADF Test shows that the data has no unit root and it is stationary.

4. Results & Discussion

Table 4.1: Descriptive Statistics of Indices

	NIFTY 100 ESG	NIFTY 50	MCSI Europe ESG Leaders	Europe STOXX 50
Mean	0.000605	0.000569	0.000228	0.000282
Median	0.001097	0.000973	0.000518	0.000512
Maximum	0.090893	0.087632	0.081142	0.092362
Minimum	-0.125822	-0.129805	-0.112153	-0.124014
Std. Dev.	0.010655	0.010850	0.009707	0.011670
Skewness	-1.157582	-1.218911	-1.013114	-0.692466
Kurtosis	23.05522	23.63054	18.56734	16.91563
Jarque-Bera	29430.09	31162.39	18740.28	14553.13
Probability	0.000000	0.000000	0.000000	0.000000
Sum	1.049109	0.985645	0.416012	0.503334
Sum Sq. Dev.	0.196621	0.203900	0.171877	0.243098
Observations	1733	1733	1825	1786

This table 4.1 presents a statistical summary of four indices: NIFTY 100 ESG, NIFTY 50, MCSI Europe ESG Leaders, and Europe STOXX 50. Among these NIFTY 100 ESG (0.000605) and NIFTY 50 (0.000569) have higher average daily returns than MCSI Europe ESG Leaders (0.000228) and Europe STOXX 50 (0.000282). Europe STOXX 50 shows the highest volatility (Std. Dev: 0.011670), followed by NIFTY 50 and NIFTY 100 ESG, while MCSI Europe ESG Leaders has the lowest of all indices.

All indices are negatively skewed, meaning they experience more small positive returns but are prone to occasional large losses. All indices exhibit high kurtosis, indicating extreme returns (both positive and negative) are more likely than in a normal distribution. The Jarque-Bera test confirms that none of the indices follow a normal distribution, as evidenced by extremely low p-values. In summary, all indices show higher volatility with the potential for extreme returns and non-normal distribution, important for understanding risk in portfolio management.

Table 4.2: Models for volatility of the NIFTY ESG 100 Index

$$Y_t = X_t{}' \theta + \epsilon_t$$

GARCH (1, 1): $\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$

GJR or TARCH (1, 1): $\sigma_t^2 = \omega + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-1}^2 + \sum_{k=1}^{r} \gamma_k \epsilon_t^2 \tau_{t-k}$

EGARCH (1, 1): $\log \sigma_t^2 = \omega + \sum_{j=1}^{q} \beta_j \log \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \left|\frac{\epsilon_{t-i}}{\sigma_{t-i}}\right| + \sum_{k=1}^{r} \gamma_k \frac{\epsilon_{t-k}}{\sigma_{t-k}}$

PARCH (1, 1): $\sigma_t^\delta = \omega + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta + \sum_{i=1}^{\rho} \alpha_i \ (|\epsilon_{t-i}| - \gamma_i \epsilon_{t-i})\delta$

IGARCH (1, 1):$\sigma_t^2 = \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2$ and $\sum_{j=1}^{q} \beta_j + \sum_{i=1}^{p} \alpha_i = 1$

	GARCH (1,1)	TGARCH (1,1)	EGARCH (1,1)	PARCH (1,1)	IGARCH (1,1)
	0.969220	0.969300	0.967684	0.969433	0.968510
θ	(0.002856)	(0.002853)	(0.002661)	(0.002851)	(0.002177)
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
				1.741605	
□				(0.435984)	
				[0.0001]	
	0.945007	0.945975	0.988660	0.945555	0.968573
□1	(0.008763)	(0.008694)	(0.003425)	(0.009146)	(0.002910)
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
	0.048177	0.055859	0.112511	0.052413	0.0311427
□1	(0.007495)	(0.011327)	(0.014656)	(0.011794)	(0.002910)
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]

$\square_1$		-0.014911 (0.013169) [0.2575]	0.011263 (0.010575) [0.2868]	-0.081672 (0.075148) [0.2771]	
R^2	0.962054	0.962052	0.962063	0.962049	0.962062
DW Stat	2.003680	2.003668	2.002508	2.003705	2.003301
Persistence	0. 945007				
Log-likelihood	8386.171	8386.698	8383.284	8387.055	8376.500
AIC	-9.672442	-9.671897	-9.667956	-9.671154	-9.663589

The table 4.2 provides estimates for different GARCH-type models (GARCH, TGARCH, EGARCH, PARCH, and IGARCH) with key parameters: θ (long-run variance), β_1 (persistence of shocks), α_1 (volatility response to shocks), γ_1 (asymmetry in volatility response), and δ (power in PARCH). The θ parameter is consistent across all models, around 0.969, indicating a high long-term volatility level across models. This suggests that these models predict substantial volatility persistence in the market.

Across all models, β_1 is close to 1, indicating high persistence in volatility. This means that shocks to volatility have long-lasting effects, especially in the EGARCH (0.988660) and IGARCH (0.968573) models, which show the highest persistence. The α_1 parameter, which measures how volatility responds to market shocks, varies across models but remains positive.

EGARCH shows the highest α_1 (0.112511), suggesting that volatility reacts more strongly to shocks in this model compared to others. IGARCH has the lowest α_1 (0.031143), indicating a relatively lower immediate volatility reaction to shocks. The γ_1 parameter, present in TGARCH and EGARCH, captures the asymmetry in volatility's reaction to positive and negative shocks. However, the estimates for γ_1 are statistically insignificant in both models, indicating that these models do not detect significant asymmetry in volatility response in this context.

The PARCH model introduces the δ parameter (1.741605), which controls the power term in volatility dynamics, capturing non-linear effects of past shocks on volatility. The significance of δ indicates that non-linear effects are important in explaining the volatility structure. All models exhibit a very similar R^2 (~0.962), meaning they explain about 96% of the variance in the data, showing that all models fit well.

Durbin-Watson statistics are close to 2, suggesting no significant autocorrelation in the residuals, a good sign for model validity. The log-likelihood values are very close across models, with PARCH (8387.055) having the highest, indicating a slightly better fit. Similarly, AIC values are close, with the GARCH model having the lowest (-9.672442), indicating it slightly edges out the others in model performance based on the Akaike criterion.

GARCH (1, 1) shows strong persistence (β_1 = 0.945007) and a good overall fit (lowest AIC). TGARCH (1, 1) introduces asymmetry in volatility, but the asymmetry effect is statistically insignificant. EGARCH (1, 1) has the highest volatility response (α_1 = 0.112511) and strong persistence (β_1 = 0.988660), but shows no significant asymmetry. PARCH (1,1) incorporates non-linear volatility effects (significant δ), performing well in terms of log-likelihood. IGARCH (1,1) shows high persistence (β_1 = 0.968573), meaning shocks have a lasting impact on volatility.

In conclusion, all models show high volatility persistence, with EGARCH and IGARCH capturing the strongest persistence, and PARCH adding the dimension of non-linear effects. The GARCH model has the lowest AIC, making it slightly preferred based on this criterion.

Table 4.3: Models for volatility of the MSCI Europe ESG Leaders

$$Y_t = X_t{}' \theta + \epsilon_t$$

GARCH (1, 1): $\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$

GJR or TARCH (1, 1): $\sigma_t^2 = \omega + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-1}^2 + \sum_{k=1}^{r} \gamma_k \epsilon_t^2 \tau_{t-k}$

EGARCH (1, 1): $\log \sigma_t^2 = \omega + \sum_{j=1}^{q} \beta_j \log \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \left|\frac{\epsilon_{t-i}}{\sigma_{t-i}}\right| + \sum_{k=1}^{r} \gamma_k \frac{\epsilon_{t-k}}{\sigma_{t-k}}$

PARCH (1, 1): $\sigma_t^\delta = \omega + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta + \sum_{i=1}^{\rho} \alpha_i \, (|\epsilon_{t-i}| - \gamma_i \epsilon_{t-i})\delta$

IGARCH (1, 1): $\sigma_t^2 = \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2$ and $\sum_{j=1}^{q} \beta_j + \sum_{i=1}^{p} \alpha_i = 1$

	GARCH (1,1)	TGARCH (1,1)	EGARCH (1,1)	PARCH (1,1)	IGARCH (1,1)
	0.745921	0.752921	0.743614	0.753106	0.748902
θ	(0.007323)	(0.007377)	(0.007716)	(0.007369)	(0.005838)
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
				2.710040	
□				(0.405026)	
				[0.0000]	
	0.912532	0.912858	0.995275	0.911726	0.928492
□1	(0.004961)	(0.005454)	(0.001324)	(0.005817)	(0.003604)
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]

$\square_1$	0.095353 (0.006782) [0.0000]	0.113283 (0.010931) [0.0000]	0.014344 (0.009799) [0.1433]	-0.095299 (0.037310) [0.0106]	0.071508 (0.003604) [0.0000]
$\square_1$			0.222238 (0.013059) [0.0000]	0.070065 (0.013953) [0.0000]	
R^2	-0.454291	-0.466918	-0.450465	-0.466405	-0.459267
DW Stat	2.164663	2.164842	2.164551	2.164854	2.164792
Persistence					
Log-likelihood	6499.871	6501.331	6492.946	6502.802	6486.149
AIC	-7.273092	-7.273607	-7.264218	-7.274134	-7.259965

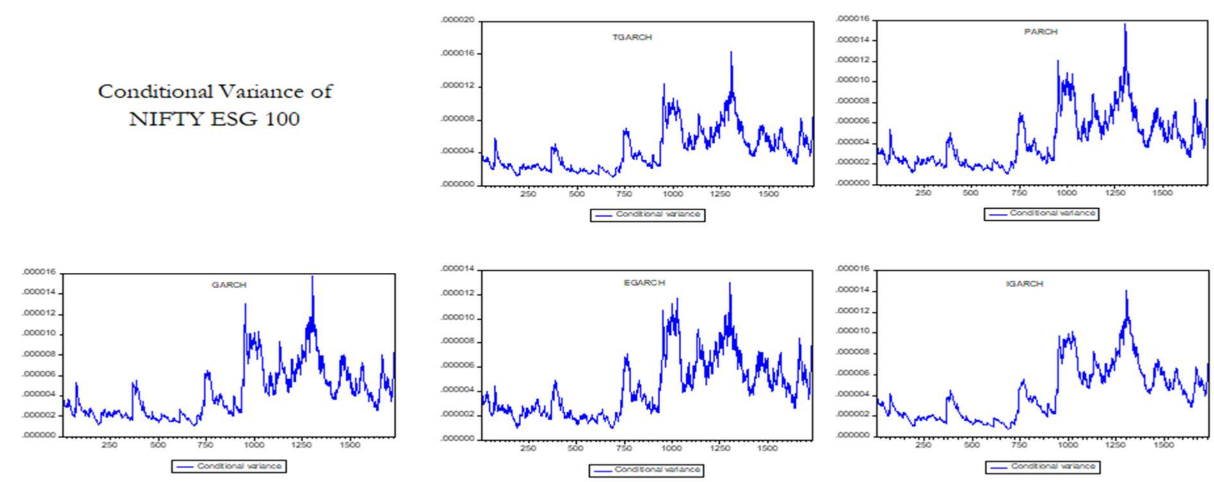
This table 4.3 summarizes the results for various GARCH-type models (GARCH, TGARCH, EGARCH, PARCH, and IGARCH), with key parameters like θ (long-run variance), β1 (shock persistence), α_1 (volatility response to shocks), γ_1 (asymmetry in volatility response), and δ (power term in PARCH). The θ parameter is similar across models, ranging from 0.7436 (EGARCH) to 0.7531 (PARCH), indicating comparable long-term volatility estimates across these models. The β_1 values are quite high across all models, showing strong persistence in volatility. The highest persistence is observed in the EGARCH model (0.995275), indicating that volatility shocks have long-lasting effects. The IGARCH model also has high persistence (0.928492), reflecting similar characteristics. The α_1 parameter, representing the response to shocks, is significant in most models. TGARCH has the highest α_1 (0.113283), showing a strong response to volatility shocks. In contrast, EGARCH has a much smaller and statistically insignificant α1 (0.014344), suggesting that this model relies more on asymmetry (γ_1) to capture volatility dynamics. PARCH has a negative α_1 (-0.095299), indicating a different response structure, while IGARCH has a lower but significant α_1 (0.071508).

The TGARCH and EGARCH models include the asymmetry parameter (γ_1). TGARCH shows a strong asymmetry (γ_1 = 0.222238), indicating that negative shocks have a larger effect on volatility than positive ones. EGARCH has a smaller but still significant asymmetry (γ_1 = 0.070065). The PARCH model introduces a power term (δ = 2.710040), capturing non-linear effects of past shocks on volatility. The significant δ indicates that the model benefits from considering non-linear volatility dynamics. All models show negative R^2 values, suggesting a poor fit in this context. A negative R^2 might arise from an inappropriate model specification or issues with the data series. Durbin-Watson statistics are all close to 2, indicating no significant autocorrelation in the residuals, which is a positive sign. The log-likelihood values are close, with PARCH (6502.802) having the highest, suggesting a slightly better model fit compared to the others. The AIC values are also similar, with PARCH having the lowest (-7.274134), indicating it slightly outperforms the other models in terms of goodness of fit based on this criterion.

GARCH (1,1) model has High persistence ($\beta_1 = 0.912532$) and significant shock response ($\alpha_1 = 0.095353$), but with a poor overall fit (negative R^2). **TGARCH (1,1)** model has Strong asymmetry ($\gamma_1 = 0.222238$) and high shock response ($\alpha_1 = 0.113283$), making it better suited for capturing volatility from negative shocks, but with a similarly poor fit. **EGARCH (1,1)** is the most persistent model ($\beta_1 = 0.995275$) with lower shock response, but notable asymmetry ($\gamma_1 = 0.070065$). However, it has a slightly worse fit than TGARCH and PARCH. **PARCH (1,1)** model introduces non-linear volatility dynamics (significant δ) and has the best fit in terms of log-likelihood and AIC, despite the negative R^2. **IGARCH (1,1)** model has High persistence ($\beta_1 = 0.928492$) and lower shock response ($\alpha_1 = 0.071508$), but overall, it shows similar limitations in model fit.

In conclusion, the models capture high volatility persistence, with TGARCH effectively modeling asymmetry and PARCH benefiting from non-linear dynamics. However, all models show poor R^2 values, suggesting that the dataset might not align well with these model structures.

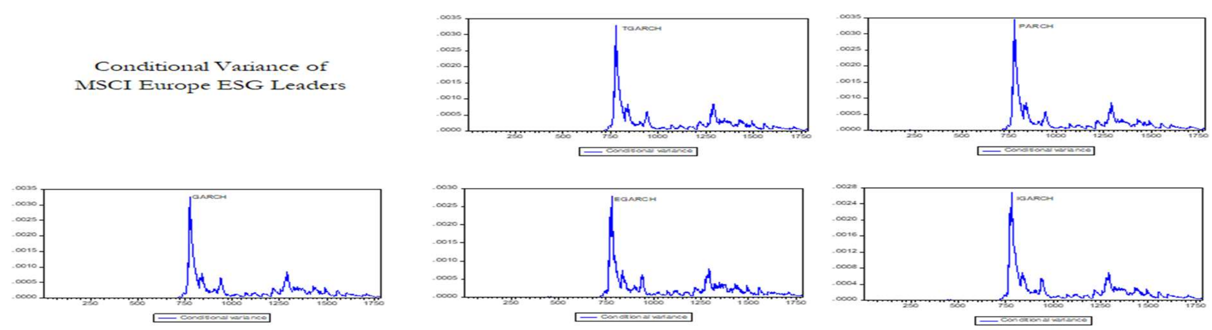
Exhibit 1: Conditional Volatility of GARCH Family Models – NIFTY ESG 100



The exhibit 1 displays the conditional variance plots of the NIFTY ESG 100 index, modeled using five different GARCH-type models: TGARCH, PARCH, GARCH, EGARCH, and IGARCH. Each plot shows how volatility evolves over time for the NIFTY ESG 100 index, with some shared observations and model-specific variations. All models demonstrate periods of increased volatility followed by mean reversion, indicating that volatility clusters, a common characteristic in financial time series. Significant volatility spikes are observed in all models, particularly in the mid to later periods, followed by gradual declines.

TGARCH Shows volatility spikes similar to the other models but with more pronounced jumps in volatility compared to GARCH. As a threshold model, TGARCH tends to capture asymmetry better, likely explaining the higher peaks in response to negative shocks. PARCH displays similar volatility dynamics to TGARCH, with several peaks over time. The non-linear nature of the PARCH model could be capturing the changes in volatility differently from the linear models, leading to slightly different dynamics in the peaks and troughs. GARCH Shows relatively smoother variance compared to the TGARCH and PARCH models but still reflects volatility clustering and significant spikes. The typical GARCH behavior of capturing long-term persistence in volatility is evident, but the model seems to underestimate sharp jumps compared to TGARCH. EGARCH exhibits high volatility spikes, similar to TGARCH, but the conditional variance seems slightly more volatile than the GARCH model. EGARCH’s ability to capture asymmetry without non-negativity constraints might explain the additional variance fluctuations during volatile periods. IGARCH shows strong persistence in volatility, with less clear mean reversion compared to the other models. IGARCH is designed to capture persistent shocks, and this is reflected in the smoother, longer-lasting periods of higher volatility across the entire time period. All models show volatility clustering and the presence of spikes in certain periods, likely driven by market events or shocks. TGARCH and EGARCH show more volatility response compared to GARCH, reflecting their ability to capture asymmetry in volatility responses to negative and positive shocks. PARCH incorporates non-linear effects, producing volatility patterns similar to TGARCH, though with a slightly different structure. IGARCH captures persistent volatility effects, showing less tendency toward volatility reversion after spikes, reflecting its core feature of assuming near-permanent effects from shocks. This comparative analysis suggests that the models capture similar volatility trends but with differences in how they react to market shocks and volatility dynamics.

Exhibit 2: Conditional Volatility of GARCH Family Models – MCSI Europe ESG Leaders



The Exhibit 2 shows the conditional variance plots for the MSCI Europe ESG Leaders index, using five GARCH-type models: TGARCH, PARCH, GARCH, EGARCH, and IGARCH. These plots reflect how each model estimates the volatility (conditional variance) over time for this index, particularly focusing on the behavior during volatile periods and periods of calm. All models indicate a significant volatility spike around the same period, followed by a sharp decline and smaller subsequent fluctuations. There is a clear clustering of volatility, with a pronounced peak that dominates the graph, suggesting a major event or market shock during this time. The variance stabilizes after the large spike, with lower but persistent fluctuations for the remainder of the period.

The TGARCH model shows a sharp and distinct peak in volatility, followed by a gradual decrease. Given that TGARCH accounts for asymmetry in volatility response (i.e., it reacts more strongly to negative shocks), the model captures a high variance during the spike, which might be tied to a period of market distress. Similar to TGARCH, PARCH shows a prominent volatility peak, but it captures the decline in volatility more gradually. The PARCH model's power term may allow it to reflect non-linear dynamics in volatility, contributing to a slightly different decay of variance after the peak compared to other models. The GARCH model demonstrates the same large spike in volatility, although the rise and fall are relatively smoother compared to the asymmetry models like TGARCH. GARCH captures persistent volatility but does not react as sharply to negative or positive shocks, making its response more symmetric. EGARCH, which also accounts for asymmetry but without the constraint that variances must be positive, shows a sharp spike similar to TGARCH, though it seems to return to lower levels of volatility faster. The model's ability to capture asymmetry without constraints might result in a more rapid decay of volatility after the initial shock. The IGARCH model displays a strong persistence in volatility, with a similar initial spike but a much more gradual decrease in conditional variance. Since IGARCH assumes permanent shocks, it shows more prolonged elevated variance following the peak, indicating that volatility shocks have longer-lasting effects in this model. All models exhibit a significant volatility spike around the same period, likely reflecting a major market event.

TGARCH and EGARCH show sharp peaks, with TGARCH displaying more sensitivity to market shocks, particularly negative ones. PARCH shows a smoother transition post-volatility, capturing non-linear effects, while GARCH displays persistent but more symmetric volatility patterns. IGARCH captures the volatility spike but assumes longer-lasting effects, reflecting slower decay in variance compared to the other models. In summary, the variance plots highlight the models' ability to capture periods of intense volatility, but with different decay rates and persistence post-shock, depending on the model structure.

5. Findings

5.1 Volatility Dynamics:

Across all models (GARCH, TGARCH, EGARCH, PARCH, and IGARCH), significant volatility spikes are observed in both indices, particularly around similar periods. For the NIFTY ESG 100, the volatility is more clustered, with repeated spikes throughout the period. In contrast, the MSCI Europe ESG Leaders index shows one pronounced spike followed by a relatively stable volatility period with smaller fluctuations. The volatility clustering evident in both indices aligns with common financial market behavior, where high volatility tends to cluster around significant events or periods of financial distress. This supports the argument that market shocks do not dissipate immediately but rather persist, creating periods of high volatility. The NIFTY ESG 100 demonstrates more frequent volatility spikes, indicating the possibility of more frequent economic events or shocks in the Indian market, while MSCI Europe ESG Leaders shows a sharp, isolated spike, suggesting a more contained but significant market event in Europe.

5.2 Leverage Effect:

The TGARCH and EGARCH models are designed to capture the leverage effect, where negative market shocks result in a higher increase in volatility than positive shocks. The conditional variance plots for these models in both indices show sharp increases in volatility, which may indicate the presence of leverage effects, particularly in response to large market downturns. The pronounced spikes in the TGARCH and EGARCH models suggest that negative shocks, such as significant sell-offs or downturns, result in disproportionate increases in volatility. This asymmetry is critical for markets like the NIFTY ESG 100 and MSCI Europe ESG Leaders, where risk perception can escalate sharply during downturns, causing volatility to rise more than during upswings. The leverage effect observed may suggest that market participants in these indices react more strongly to bad news, increasing volatility following negative market movements.

5.3 Asymmetric Effect:

The TGARCH and EGARCH models show asymmetry in volatility response to positive and negative shocks, which is more pronounced for NIFTY ESG 100 than for MSCI Europe ESG Leaders. For instance, the conditional variance in the TGARCH models shows larger volatility spikes, indicating that the market reacts more strongly to negative information or events. The asymmetric effect is an important finding as it demonstrates that negative news or events tend to have a larger impact on market volatility compared to positive news. In the NIFTY ESG 100, the frequent and significant spikes in volatility suggest that the Indian market is more sensitive to asymmetric shocks, potentially due to higher market speculation or less mature market structures. For the MSCI Europe ESG Leaders, the asymmetry appears more isolated, indicating that while negative events do lead to increased volatility, the market tends to recover more steadily compared to the Indian index. This may reflect differences in market depth, regulatory environment, and the investor base between these two regions.

5.4 Spillover Effect:

The prolonged periods of elevated volatility in the IGARCH models, especially for MSCI Europe ESG Leaders, indicate that volatility shocks have long-lasting effects, even after the initial shock has passed. The persistence of volatility in the IGARCH model suggests that volatility spillover may be a factor, particularly in global indices like MSCI Europe ESG Leaders, where volatility from one market can spill over into others. Volatility spillover is the transmission of volatility from one market to another, and it is more likely in globally interconnected markets. The IGARCH model's persistent volatility in MSCI Europe ESG Leaders may indicate that volatility in one European country or sector could spread across the region, keeping the overall volatility elevated. In contrast, the NIFTY ESG 100 shows more localized spikes in volatility, suggesting less pronounced spillover effects. This could be due to the relatively more insulated nature of the

Indian market compared to Europe, where markets are more integrated and susceptible to global events.

The findings from the conditional variance plots across the GARCH-type models highlight important features of volatility, including the leverage and asymmetric effects, as well as the potential for volatility spillover. The Indian market (NIFTY ESG 100) shows more frequent and sharper volatility clusters, with pronounced leverage and asymmetric effects. On the other hand, the European market (MSCI Europe ESG Leaders) experiences fewer but more significant volatility spikes, with prolonged persistence in volatility, possibly due to spillover effects from interconnected global markets. Understanding these dynamics is crucial for investors and policymakers to effectively manage risk, especially in periods of financial instability. By incorporating models like TGARCH and EGARCH, market participants can better anticipate asymmetric volatility reactions, while IGARCH can provide insights into the long-term persistence of volatility, allowing for more informed hedging and risk management strategies.

6. Suggestions

While the current models (GARCH, TGARCH, EGARCH, PARCH, and IGARCH) provide valuable insights into volatility, adding more sophisticated models such as Component GARCH (CGARCH) or Multivariate GARCH (MGARCH) could offer a more granular understanding of long-term and short-term volatility components. MGARCH models, in particular, can account for the co-movement between multiple assets or indices, which could be beneficial for analyzing the spillover effect between different markets (e.g., between Europe and India).

The current models are based on daily or monthly observations. By utilizing high-frequency intraday data, the models could capture intraday volatility patterns, leading to a more accurate reflection of market movements and more responsive risk management strategies. This is particularly important for identifying early signs of volatility shocks and managing them in real-time.

The EGARCH and TGARCH models capture the asymmetric effects well, but further improvement can be achieved by introducing models like Threshold Autoregressive Conditional Duration (TACD) or using Nonlinear GARCH models. These alternatives are designed to more accurately capture volatility behavior in response to large shocks or structural breaks, which may help refine the understanding of volatility spikes in both markets.

Since the IGARCH model reveals long-lasting volatility persistence, improving volatility forecasting by using machine learning algorithms (e.g., Long Short-Term Memory (LSTM) models or Random Forests) could enhance the prediction accuracy of future market volatility. Additionally, implementing spillover management strategies, such as cross-hedging or diversification across low-correlated assets, can help mitigate the risks associated with volatility spillovers in global indices like MSCI Europe ESG Leaders.

To complement the volatility models, incorporating macro-economic indicators (like inflation, interest rates, or geopolitical risk) or global risk factors (like oil prices or exchange rates) can provide a more comprehensive understanding of the volatility dynamics. A GARCH-M (GARCH-in-Mean) model could be used to explicitly include economic variables and better capture the risk-return tradeoff.

The findings highlight significant volatility spikes that are likely driven by major market events. Applying stress testing or scenario analysis to model the impact of extreme market conditions could provide a clearer picture of how both markets might react to future extreme shocks. This would help investors, particularly those in ESG indices, prepare for unforeseen risks.

Given the strong leverage and asymmetric effects, especially in the NIFTY ESG 100, it's recommended to develop risk management tools that specifically account for these effects. For example, dynamic hedging strategies or option-based risk management tools that are sensitive to negative market shocks could be developed to manage portfolio risks more effectively.

Since these indices are ESG (Environmental, Social, and Governance) based, it may be beneficial to explore how ESG factors influence volatility. Investigating whether ESG-related events (such as regulatory changes, corporate governance issues, or environmental crises) have a direct impact on volatility could provide deeper insights into the risk-return dynamics of ESG portfolios.

Incorporating these suggestions can enhance the modeling of volatility and improve risk management strategies. The introduction of more advanced GARCH models, machine learning techniques, and economic indicators could lead to better predictions and a more comprehensive understanding of volatility dynamics in both the NIFTY ESG 100 and MSCI Europe ESG Leaders indices. This would also enable investors and policymakers to develop more robust strategies for managing risks, especially during periods of market stress and global uncertainty.

7. Conclusion

The analysis of volatility dynamics for both NIFTY ESG 100 and MSCI Europe ESG Leaders using various GARCH-type models highlights key patterns in volatility clustering, leverage effects, asymmetric responses, and the persistence of volatility. Both indices exhibit significant volatility spikes, with the NIFTY ESG 100 showing more frequent volatility clusters, suggesting a more dynamic risk environment compared to the MSCI Europe ESG Leaders, which experiences fewer but sharper volatility spikes. The presence of leverage and asymmetric effects, particularly in response to negative market shocks, underscores the importance of accounting for these factors in volatility modeling and risk management.

The spillover effect, more evident in the MSCI Europe ESG Leaders index, suggests the interconnectedness of global markets and the potential for volatility in one region to influence others. The persistent volatility in the IGARCH model further emphasizes the need for robust hedging strategies that can mitigate long-term volatility risks.

Moving forward, integrating more sophisticated volatility models, high-frequency data, and economic factors into the analysis can enhance forecasting accuracy and provide deeper insights into the drivers of market volatility. Furthermore, leveraging machine learning techniques and stress testing under extreme scenarios can strengthen risk management frameworks, helping investors and policymakers make informed decisions in an increasingly volatile global financial environment. By understanding these volatility patterns and their underlying causes, better hedging strategies can be developed, ensuring more stable and resilient portfolios, especially in ESG-focused investments.

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