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DIFFERENCE SQUARE MEAN FUZZY LABELLING OF TREE RELATED GRAPHS

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Abstract:

A graph is said to be a mean square fuzzy graph if it is characterized by the number of edges that are the least significant of any two neighboring vertices. In this study, we focus on the mean square fuzzy labeling technique for newly constructed graphs generated by replacing each edge of balloon trees, shrub trees, banana trees, and coconut trees. These are examples of tree graphs.

KeyWords:

Balloon tree, Shrubtree , Banana Tree, Coconut tree.

Introduction:

In graph theory, graph labeling is a well-known academic topic. A sizable number of open issues and literature exist for different kinds of graphs. In the 1960s, Rosa presented the graph labeling hypothesis. Along with graph labeling, fuzzy labeling is the most intriguing type, with a wide range of uses [1,2,4].Assigning labels to a graph's edges and/or vertices typically represented by integers is known as graph labeling in the mathematical field of graph theory [3,14].

One particular kind of structure that is linked and acyclic that is, devoid of loops or cycles is a tree. It is made up of nodes, or vertices, and edges, or connections between nodes, with each pair of nodes having precisely one path. Stated differently, a tree is a graph in which there are no circular paths and you can move between any two nodes without retracing any edges [11,12,15].

The fuzzy set was first presented by Zadeh as a class of object with a number of membership degrees. The fuzzy approach includes to a degree of membership between [0,1], which can be explained based on of the membership function of a fuzzy number, as opposed to traditional crisp sets, which are characterized by either membership or non-membership[7,8].

Zadeh was the first to define a fuzzy relation on a set in 1965. Kaufmann presented the first definition of a fuzzy graph in 1973, while Azriel Rosenfeld established the structure of fuzzy graphs in 1975. There are numerous further uses for fuzzy graphs in the modeling of real-time systems with varying degrees of precision and information inherent in the system. Rosa first proposed the idea of graceful labeling in 1966 [10]

Basic Definitions:

1. **Definition:**

A unique type of graph known as a tree has no cycles, but all of its vertices are attached. It implies that every path from one vertex to another will be distinct and won't return to any earlier location. To create a linked graph that resembles a tree, a graph with n vertices must have n – 1 edges.

2. **Definition:**

A fuzzy graph is a pair G = (η, γ) where η is a fuzzy subset of V and γ is a fuzzy relation on η such that γ (u,v) ≤ η (u) ∧ η (v),for all u,v ∈ V.

3. **Definition:**

A graph $G = (V, E)$ is said to be a difference square mean fuzzy labelling if $\eta: V \rightarrow [0,1]$ and $\gamma: VxV \rightarrow [0,1]$ are bijective and $\gamma(u,v) < \eta(u) \wedge \eta(v)$ and $\gamma(u,v) = ||\eta(u - \eta(v)|^2/2|$ Fore very $(u,v) \in E$ are all distinct. A graph which that satisfies called Difference square mean fuzzy labelling.

4. **Definition [5]:**

To create the (n, m) balloon tree graph, join a single edge from each of the n prints of the m-star graph. The balloon tree graph is represented by BTn,m.

5. **Definition:[13]**

A network called Tree ST (n Δ (2,3,…n)) is created by joining a vertex to the central vertex of every number of stars.

6. **Definition [9]:**

A graph of banana trees By joining a vertex v with m pathways and M paths with n paths, a graph known as BNT (K1Δ m,mΔn) is created.

7. **Definition [6] :**

The graph of a coconut tree CT(m,n) is obtained by attaching "n" new pendant points at the terminal vertex of Pm for every positive integer n and m ≥ 2.

Theorem: 1

The Balloon Tree Graph $BT_{n,m}$ is difference square mean fuzzy graph.

Proof:

$BT_{n,m}$ be Balloon Tree Graph. The definition of the given graph, n number of vertices attached m number of vertices. The tree graph $g = i + n + nm$ and $h = nm + n$

Then vertex set are $\eta(G) = \{v, v_1, v_2, \dots v_n, u_1, u_2, \dots . u_m\}$ where v vertex attached by the vertices are $\{v_1, v_2, \dots v_n\}$ and $\{v_1, v_2, \dots v_n\}$ are the vertices attaching by the vertices are $\{u_1, u_2, \dots . u_n\}$ as shown figure(1)

The edge set $\gamma(G) = \{e_1, e_2, \dots . e_n, e'_1, e'_2, \dots . e'_m\}$ where $e_i = \{v, v_i\}, e'_{ij} = v_i, u_j\}$

Define the DSMF graph function

$\eta(v) = 0.001$

$\eta(v_i) = (g + h - 2n + 2i)0.001$ For i=1,2,….n

$\eta(v_{ij}) = (2j + 2i + (i-1)2m)0.001$ For i=1,2,….n

For j=1,2,….n

Then the edge labels are

$\gamma(e_i) = [[(g + h + -2n + 2i) - 1]0.001]^2/2$

$\gamma(e_{ij}) = [|(2j + (i-1)2m - g - h + 2n)|0.001]^2/2$

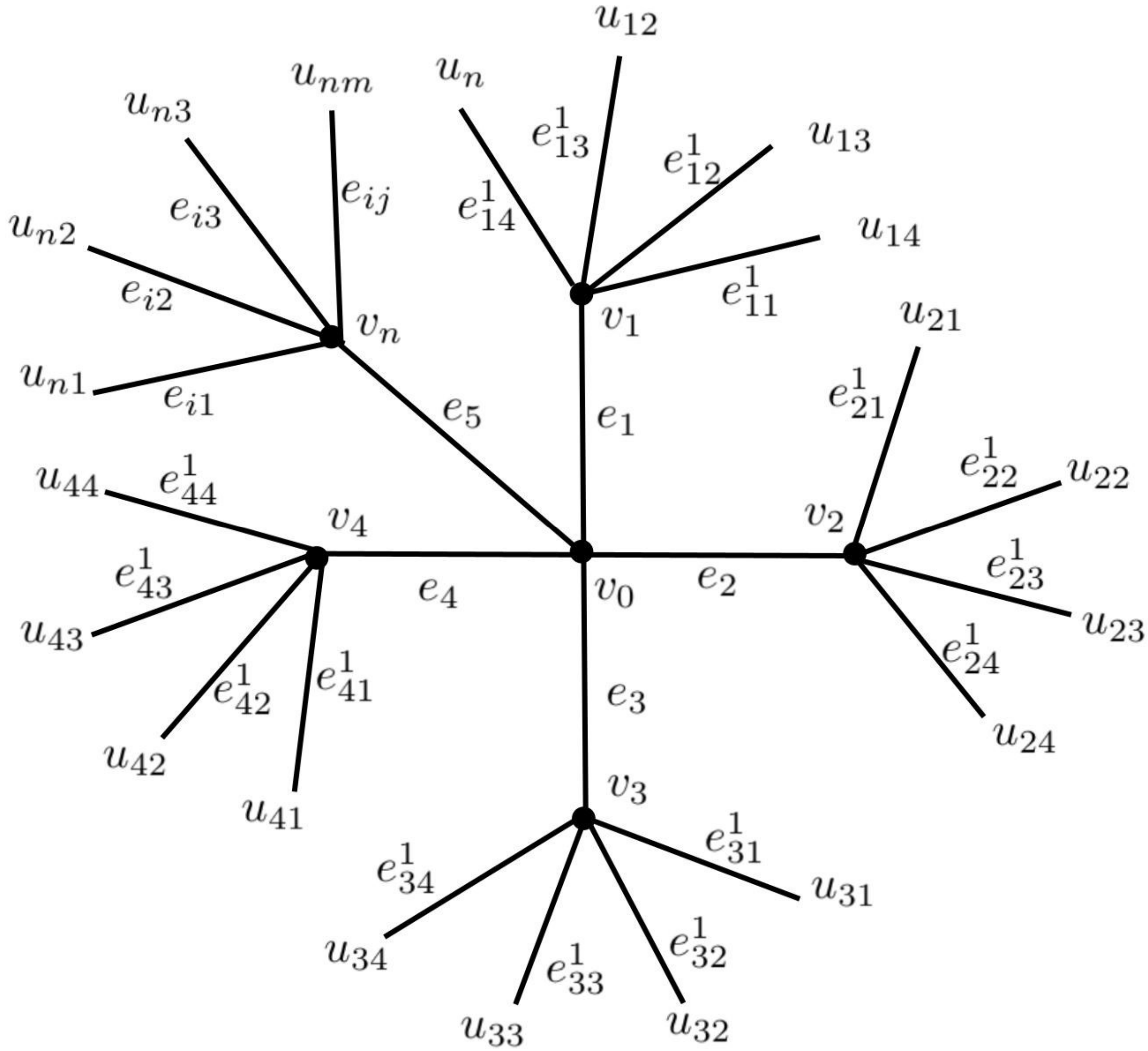

Fig1: Difference square mean Fuzzy labelling of $BT_{(n,m)}$

Hence, the $BT_{(n,m)}$ be the difference square mean fuzzy graph.

Example:

The graph $BT_{4,3}$ is shown fig(2). Balloon tree, $g = 17$ and $h = 16$

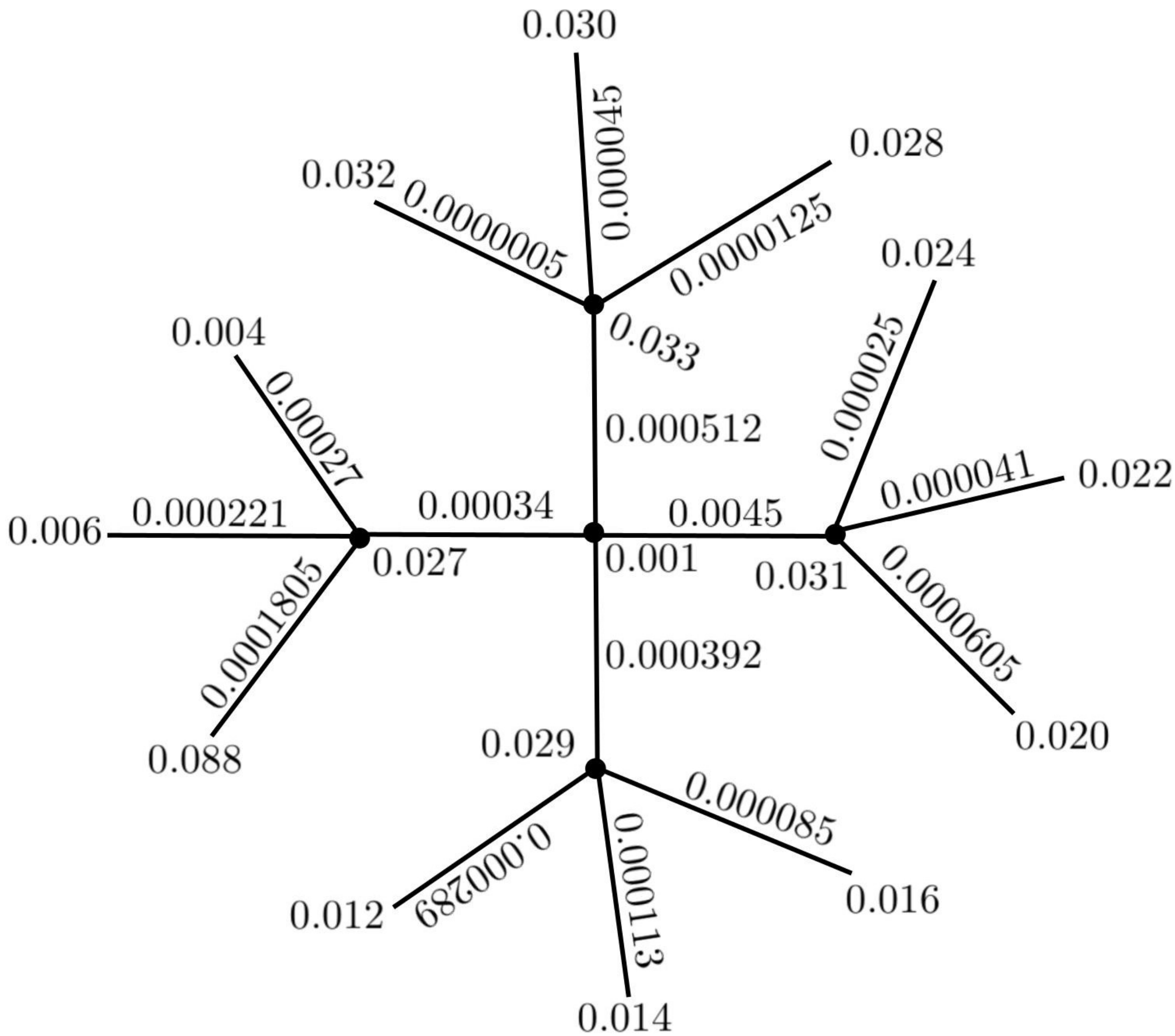

Fig: 2 DSMFL of $BT_{4,3}$

The vertex labels are

$\eta(v) = 0.001$

$\eta(v_1) = 0.033$	$\eta(v_3) = 0.029$
$\eta(v_2) = 0.031$	$\eta(v_4) = 0.027$

$\eta(u_{11}) = 0.032$	$\eta(u_{21}) = 0.024$
$\eta(u_{12}) = 0.030$	$\eta(u_{22}) = 0.022$
$\eta(u_{13}) = 0.028$	$\eta(u_{23}) = 0.020$

$\eta(u_{31}) = 0.016$	$\eta(u_{41}) = 0.008$
$\eta(u_{32}) = 0.014$	$\eta(u_{42}) = 0.006$
$\eta(u_{33}) = 0.012$	$\eta(u_{43}) = 0.004$

The Edge Labels are

$\gamma(e_1) = 0.000512$ $\gamma(e_3) = 0.000392$
$\gamma(e_2) = 0.00045$ $\gamma(e_4) = 0.00034$

$\gamma(e'_{11}) = 0.0000125$ $\gamma(e'_{21}) = 0.000025$
$\gamma(e'_{12}) = 0.000045$ $\gamma(e'_{22}) = 0.000041$
$\gamma(e'_{13}) = 0.000005$ $\gamma(e'_{23}) = 0.000061$
$\gamma(e'_{31}) = 0.000085$ $\gamma(e'_{41}) = 0.0001805$
$\gamma(e'_{32}) = 0.000113$ $\gamma(e'_{42}) = 0.0000221$
$\gamma(e'_{33}) = 0.000289$ $\gamma(e'_{43}) = 0.00027$

Hence $BT_{4,3}$ is Difference square mean fuzzy labelling graph.

Theorem: 2

The shrub tree graph $ST_n \odot (2,3,\ldots.n+1)$ is difference square mean fuzzy graph.

Proof:

$ST_n \odot (1,2,3,\ldots.n+1)$ be a shrub free graph. The Definition of the given graph , n number of vertices attached $(2,3,4 \ldots n+1)$ vertices. Then the graph point $g = 7(n-1)-2$ and distance $h = 7(n-1)-1$

The vertex set are $\eta(G) = \{u, u_1, u_2 \ldots . u_n, v_1, v_2, v_3, \ldots .. v_1, v_2, v_3, \ldots v_{6(n-1)-4}\}$ where u vertex attached by the vertices are $\{u, u_1, u_2 \ldots . u_n\}$ and $\{u, u_1, u_2 \ldots . u_n\}$ are the vertices attaching by the vertices are $\{v_1, v_2 \ldots . v_{6(n-1)-4}\}$

The edge set $\gamma(G) = \{e_1, e_2, \ldots . e_n, f_1, f_2, \ldots .. f_{6(n-1)-4}\}$ where $e_i = \{u, u_1\}$, $f_i = \{u_i, v_{6(n-1)-4}\}$

Define the DSMF graph function.

Vertex Labels

$\eta(u) = 0.001$
$\eta(u_i) = (i+1)0.001$ for $1 \le i \le n$
$\eta(v_{2i-1}) = (4+2i)0.001,$ for $1 \le i \le 2n-1,$
$\eta(v_{2i}) = (4+2i+1)0.001,$ for $1 \le i \le 2n-1,$

Edge Labels

$\gamma(e_i) = [|(i)0.001|^2]/2$ $1 \le i \le n$
$\gamma(f_{2i+1}) = [|(3+i)0.001|^2]/2$ $1 \le i \le 2n-1$
$\gamma(f_{2i}) = [|(4+i)0.001|^2]/2$ $1 \le i \le 2n-1$

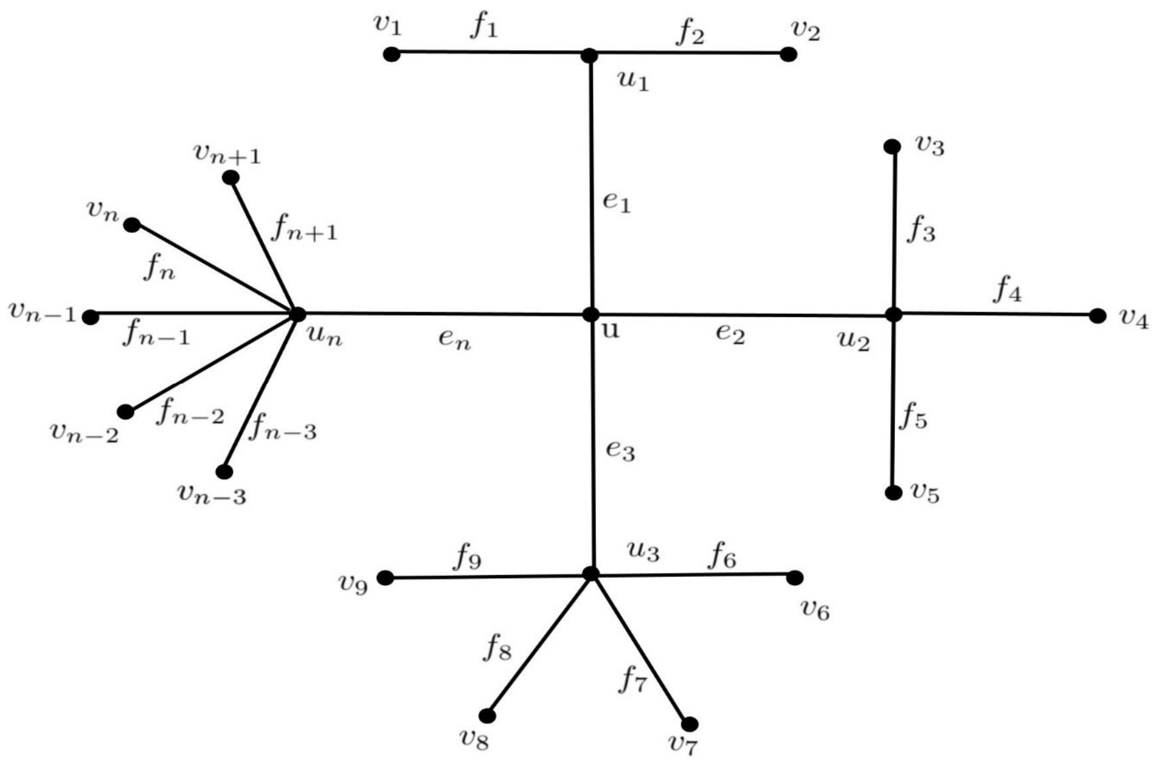

Fig.3 Difference square mean fuzzy labelling $ST_n \odot (2,3,4,\ldots.n+1)$

Hence the $ST_n \odot (2,3,4,\ldots.n+1)$ be the difference square mean fuzzy graph.

Example:

The graph $ST_4 \odot (2,3,4,5)$ as shown in Fig(4) shrub tree, $g = 19$ and $n = 18$

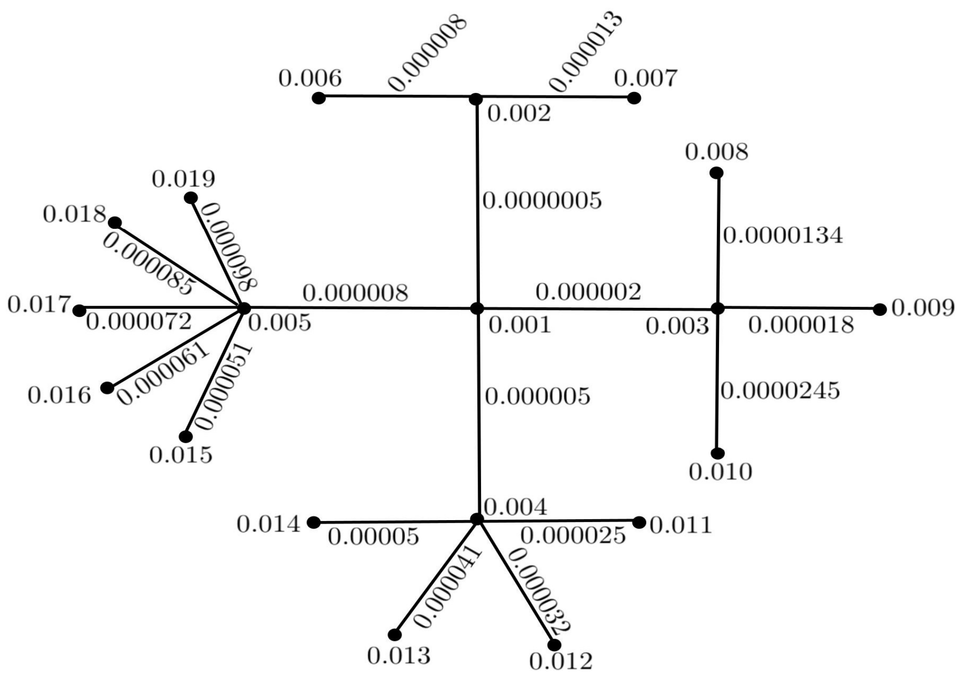

Fig.4 DSML of $ST_n \odot (2,3,4,5)$

Vertex Labels

$\eta(u) = 0.001$

$\eta(u_1) = 0.002$, $\eta(u_2) = 0.003$,

$\eta(u_3) = 0.004$, $\eta(u_4) = 0.005$

$\eta(v_1) = 0.006$, $\eta(v_2) = 0.007$

$\eta(v_3) = 0.008$ $\eta(v_4) = 0.009$

$\eta(v_5) = 0.010$ $\eta(v_6) = 0.011$

$\eta(v_7) = 0.012$ $\eta(v_8) = 0.013$

$\eta(v_9) = 0.014$ $\eta(v_{10}) = 0.015$

$\eta(v_{11}) = 0.0016$ $\eta(v_{12}) = 0.017$

$\eta(v_{13}) = 0.018$ $\eta(v_{14}) = 0.019$

Edge Labels

$\gamma(e_1) = 0.000005$ $\gamma(e_3) = 0.000005$

$\gamma(e_2) = 0.000002$ $\gamma(e_4) = 0.000008$

$\gamma(f_1) = 0.000008$ $\gamma(f_3) = 0.0000134$

$\gamma(f_2) = 0.000013$ $\gamma(f_4) = 0.000018$

$\gamma(f_5) = 0.0000245$

$\gamma(f_6) = 0.000025$ $\gamma(f_{10}) = 0.000051$

$\gamma(f_7) = 0.000032$ $\gamma(f_{11}) = 0.000061$

$\gamma(f_8) = 0.000041$ $\gamma(f_{12}) = 0.000072$

$\gamma(f_9) = 0.00005$

$\gamma(f_{13}) = 0.000085$

$\gamma(f_{14}) = 0.000098$

Hence, Shrub Tree (4⊙(2,3,4,5) is Difference square mean labelling graph.

Theorem: 3

Banana tree graph $BNT(k_1 \odot m, m \odot n)$ is difference square mean fuzzy graph.

Proof:

Let $BNT(k_1 \odot n, n \odot m)$ be a banana tree of point $g = (n+1)4m - 2$ and distance $h = (n+1)4m - 1$. The vertex set $\eta(G) = \{v, v_1, v_2, \dots v_m, u, u_1, u_2, \dots . u_m, w, w_1, w_2, \dots . w_n\}$ where v is the centre of the vertex. Vertex v attached by the vertices are $\{v_1, v_2, v_3, \dots . v_m\}$. and $\{v_1, v_2, v_3, \dots . v_m\}$ are the vertices attached by the vertices are $\{u_1, u_2, \dots . u_m\}$. The vertices are $\{u_1, u_2, \dots . u_n\}$. attached by the vertices are $\{w_1, w_2, \dots . w_n\}$.

The Edge set $\gamma(G) = \{c_1, c_2, c_3, \dots c_m, e_1, e_2, e_3, \dots . e_n, f_1^1, f_2^1, f_3^1, \dots . f_n^2, f_1^2, f_2^2, f_3^2, \dots . f_n^2\}$ where $c_i = \{c, c_i\}$, $e_i = \{v_i, u_i\}$ $f_i = \{u_i, w_i\}$

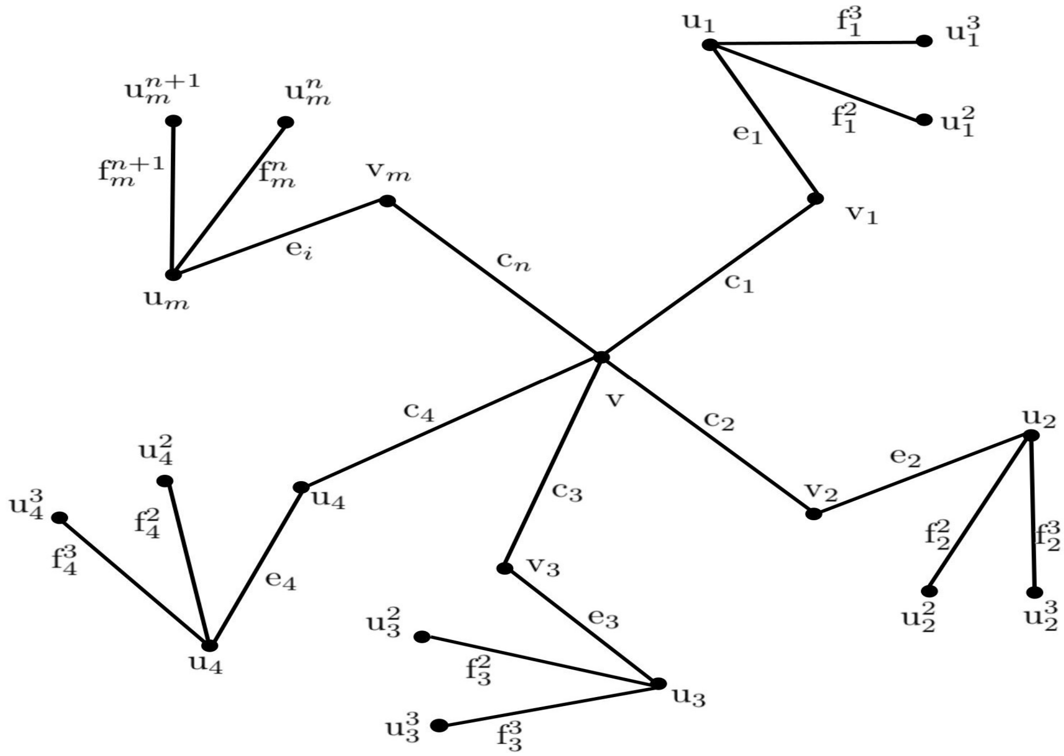

Fig.5 DSML of $BNT(k_1 \odot m, m \odot n)$

Vertex Labels

$\eta(v) = (4m(n+1) - 2)0.001$ For $1 \leq i \leq m$

$\eta(v_i) = [4(i-1)]0.001$ For $1 \leq i \leq m$

$\eta(u_i) = [8(i-1) + 4m(n-1) + 2]0.001$ For $1 \leq i \leq m$

$\eta(u_i^j) = [4(i-1) + 4m(j-)]0.001$ For $1 \leq i \leq m$

$2 \leq i \leq n$

Edge Labels

$\gamma(c_i) = [|[4m(n+1) + 2 - 4i]|^2 0.001]/2,$ $1 \leq i \leq m$

$\gamma(e_i) = |[4[i-1] + 4m(n-1) + 2]|^2 * 0.001]/2,$ $1 \leq i \leq m$

$\gamma(f_i^j) = |4m(n+1-j) + 4i - 6|^2 0.001/2,$ $1 \leq i \leq m$

$2 \leq j \leq n$

Hence, Banana tree graph, has a difference square mean fuzzy labelling graph.

Example:4

The banana tree graph $(k_1 \odot 4, 4(1,2,3))$ is shown in figure 8. $g = 17$, $h = 16$.

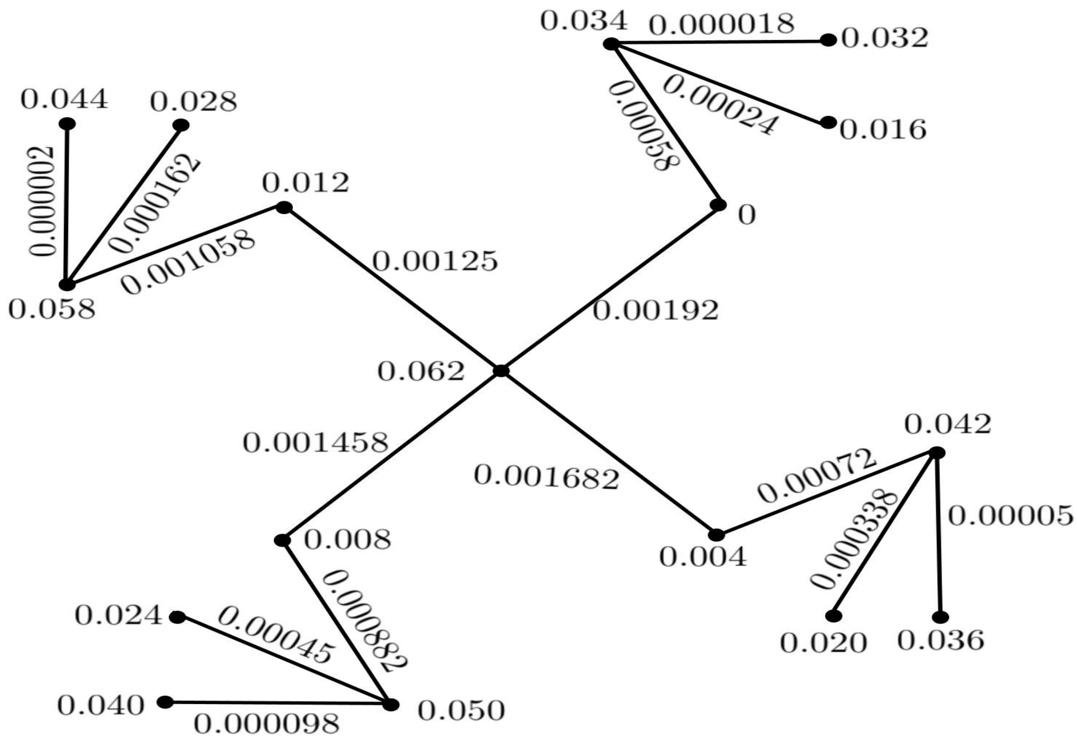

Fig.6 DSML of $BNT(k_1 \odot 4,4(1,2,3))$

Vertex Labels

$\eta(v) = 0.062$

$\eta(v_1) = 0$ $\eta(v_2) = 0.004$

$\eta(v_3) = 0.008$ $\eta(v_4) = 0.012$

$\eta(u_1) = 0.034$ $\eta(u_2) = 0.042$

$\eta(u_3) = 0.050$ $\eta(u_4) = 0.058$

$\eta(u_1^2) = 0.016$ $\eta(u_2^2) = 0.020$ $\eta(u_1^3) = 0.032$ $\eta(u_2^3) = 0.036$

$\eta(u_3^2) = 0.024$ $\eta(u_4^2) = 0.028$ $\eta(u_3^3) = 0.040$ $\eta(u_4^3) = 0.044$

Edge Labels

$\gamma(c_1) = 0.001922$ $\gamma(e_1) = 0.00058$

$\gamma(c_2) = 0.001682$ $\gamma(e_2) = 0.00072$

$\gamma(c_3) = 0.001458$ $\gamma(e_3) = 0.000882$

$\gamma(c_4) = 0.00125$ $\gamma(e_4) = 0.001058$

$\gamma(f_1^2) = 0.000162$ $\gamma(f_1^3) = 0.000002$

$\gamma(f_2^2) = 0.000242$ $\gamma(f_2^3) = 0.000018$

$\gamma(f_3^2) = 0.000338$ $\gamma(f_3^3) = 0.00005$

$\gamma(f_4^2) = 0.00045$ $\gamma(f_4^3) = 0.000098$

Theorem: 4

Coconut Tree $CT_{m,n}$ is Difference square mean fuzzy labelling graph.

Proof:

Let $CT_{m,n}$ be the coconut tree graph of point $g = m + n$ distance $h = m + n - 1$

The vertex set $\eta(G) = \{v, v_1, v_2, \dots v_n, u_1, u_2, \dots . u_m\}$ where v is the centre of the joint vertex, The vertex v attached by the vertices are $\{v_1, v_2, \dots v_n\}$ and the vertex v attached by the vertices are $\{u_1, u_2, \ldots . u_m\}$. Then, the edge set $\eta(G) = \{e_1, e_2, \ldots . e_n, f_{1,} f_{2,} \ldots f_m\}$ where $e_{i,} = \{v_1 v_i\}$, $f_{i,} = \{v_1 u_i\}$ for $1 \leq i \leq n$ and $1 \leq i \leq m$.

The Vertex Label

$\eta(v) = 0.001$

$\eta(v_{2i}) = (2i+1)0.001$ for $1 \leq i \leq n - 1/2$ if n is odd

$1 \leq i \leq n/2$ if n is even

$\eta(v_{2i-1}) = (2n - 2i + 1)0.001$ for $1 \leq i \leq n/2$ if n is odd

$1 \leq i \leq n - 1/2$ if n is even

$\eta(u_i) = (2n + 2i - 1)0.001$ for $1 \leq i \leq m$.

The Edge label

$\gamma(e_{2i-1}) = [|(2n - 4i)|^2/2]0.001$ for $1 \leq i \leq n/2$ if n is odd

$1 \leq i \leq n - 1/2$ if n is even

$\gamma(e_{2i}) = [|(4i - 2n)|^2/2]0.001$ for $1 \leq i \leq n - 1/2$ if n is odd

$1 \leq i \leq n/2$ if n is even

$\gamma(f_i) = [|2n + 2i|^2/2]0.001$ for $1 \leq i \leq m$

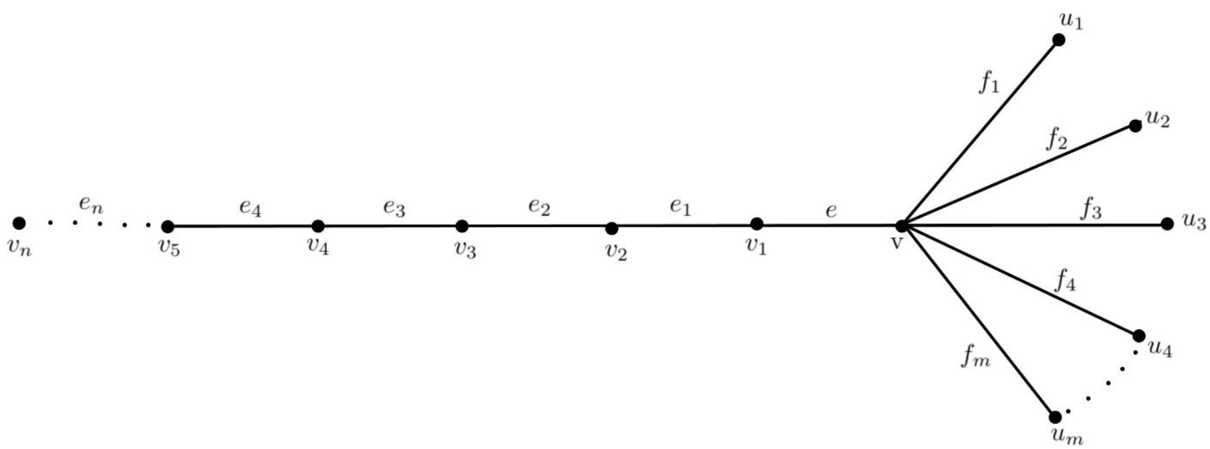

Fig.9 Generalized graph of $CT(mn)$

Hence coconut graph has a difference square mean fuzzy labelling graph.

Example:

The $CT(6,4)$, and $CT(5,7)$ is shown in Fig.10 $CT(6,4)$, point $g = 10$ distance $h = 9$

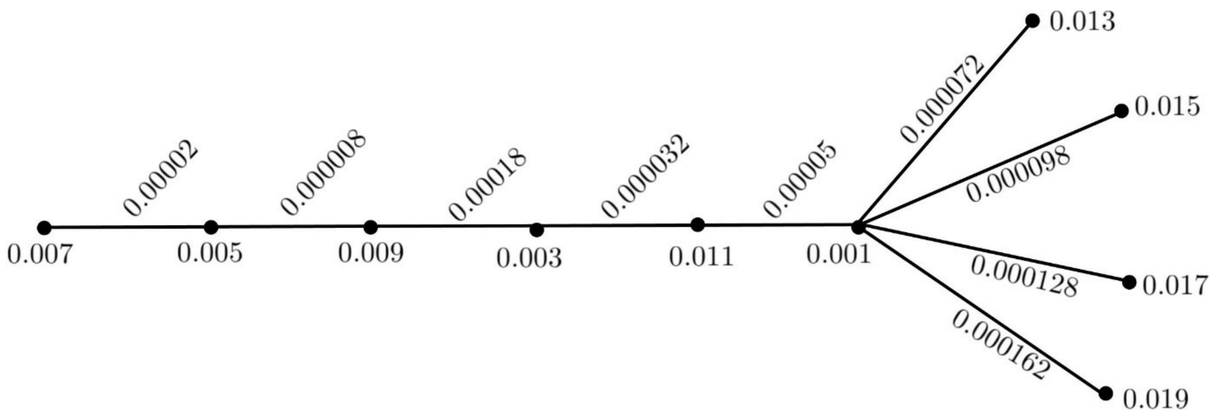

Fig.7 Coconut Tree graph of $CT(6,4)$ is DSMFL.

Vertex Label
$\gamma(v) = 0.001$
$\eta(v_1) = 0.0011$ $\eta(u_1) = 0.013$
$\eta(v_2) = 0.003$ $\eta(u_2) = 0.015$
$\eta(v_3) = 0.009$ $\eta(u_3) = 0.017$
$\eta(v_4) = 0.005$ $\eta(u_1) = 0.019$
$\eta(v_5) = 0.007$
Edge Labels
$\gamma(e) = 0.00005$ $\gamma(f_1) = 0.000072$
$\gamma(e_1) = 0.000032$ $\gamma(f_2) = 0.000098$
$\gamma(e_2) = 0.00018$ $\gamma(f_3) = 0.000128$
$\gamma(e_3) = 0.000008$ $\gamma(f_4) = 0.000162$
$\gamma(e_4) = 0.00002$

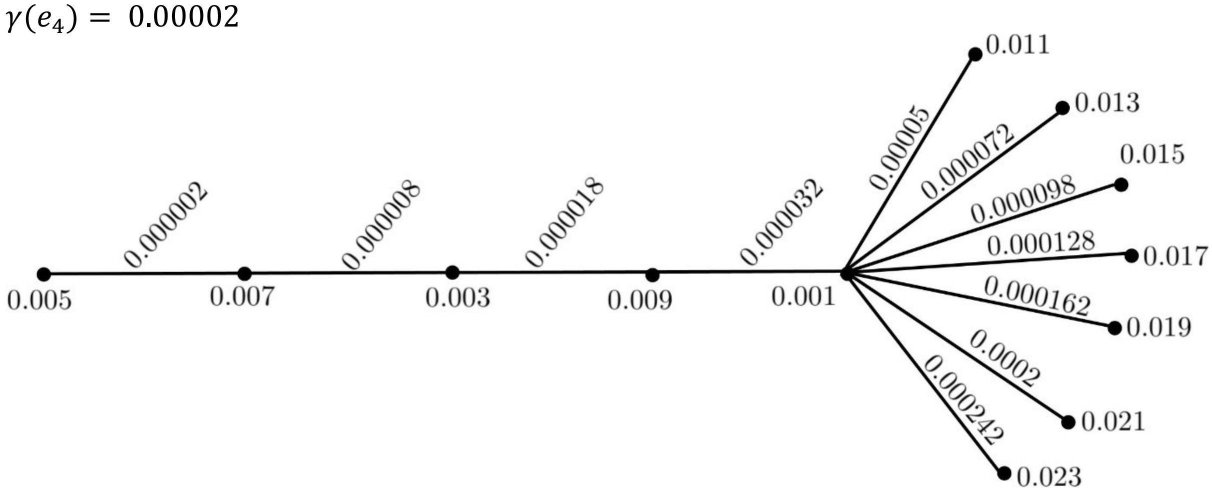

Fig.8 Coconut Tree graph of $CT(5,7)$ is DSMFL

Vertex Label
$\eta(v) = 0.001$
$\eta(v_1) = 0.009$ $\eta(u_1) = 0.011$
$\eta(v_2) = 0.003$ $\eta(u_2) = 0.013$

$\eta(v_3) = 0.007$ $\eta(u_3) = 0.015$
$\eta(v_4) = 0.005$ $\eta(u_4) = 0.017$
$\eta(u_5) = 0.019$
$\eta(u_6) = 0.021$
$\eta(u_7) = 0.023$
Edge Labels
$\gamma(e) = 0.000032$ $\gamma(f_1) = 0.00005$
$\gamma(e_1) = 0.000018$ $\gamma(f_2) = 0.000072$
$\gamma(e_2) = 0.000008$ $\gamma(f_3) = 0.000098$
$\gamma(e_3) = 0.000002$ $\gamma(f_4) = 0.0000128$
$\gamma(f_5) = 0.000162$ $\gamma(f_6) = 0.0002$
$\gamma(f_7) = 0.000242$

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