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# LINGUISTIC DUAL HESITANT SUPER HYPERSOFT TOPOLOGICAL SPACE IN MULTI CRITERIA DECISION MAKING

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**Abstract**

In this article presents a topology-based approach using linguistic dual hesitant super hypersoft sets to support medical diagnosis under conditions of vagueness and multiple interacting attributes. The method draws on dual hesitant membership and non-membership values along with topological notions, subbase, interior, and closure to structure and refine the evaluation process. A decision algorithm is proposed that merges the linguistic dual hesitant super hypersoft set assessments with these topological operations to rank alternatives and support decision-making.

**Keywords:** linguistic set, dual hesitant set, hypersoft set, super hypersoft set, super hypersoft topology, multi criteria decision making.

**1. Introduction:** Different degrees of uncertainty have been addressed by fuzzy sets through the incorporation of possible membership and possible non-membership values. Ambiguity, imprecision, and vagueness are inherently exhibited by linguistic formulations. However, when hierarchical parameter structures and decision problems involving super hypersoft attribute families are encountered, limitations are shown by these models.

Hypersoft sets were generalized through the proposal of super hypersoft sets, in which super hypersoft attribute family functions were extended. A flexible framework for the modelling of membership and non-membership across super hypersoft attribute families is provided when linguistic dual hesitant sets are combined with this concept, resulting in linguistic dual hesitant super hypersoft sets. Moreover, the structural analysis of such sets is further enhanced through the introduction of linguistic dual hesitant super hypersoft topology, by which open sets are defined and topological operations are made possible within a linguistic dual hesitant super hypersoft set environment.

**Motivation and research gap:** The motivation and research gap for this article is driven by the growing complexity that has come to characterize medical diagnostic processes, in which decisions must be reached based on information that is heterogeneous, uncertain, and, at times, contradictory. In many real-world clinical situations, diseases are not clearly indicated by symptoms, inconclusive results are often yielded by tests, and variation is exhibited by the super hypersoft attribute family from case to case.

A natural mathematical structure through which these challenges can be addressed is provided by linguistic dual hesitant super hypersoft sets, as independent modeling of possible membership and possible non-membership across combinations of medical attributes is thereby allowed. However, a topological perspective is lacking in most existing linguistic dual hesitant based decision models, even though valuable insights into the structural relationships among diagnostic alternatives could be offered by such a perspective. Through the introduction of a linguistic dual hesitant super hypersoft topology, medical decisions can be analyzed by means of open sets and topological operators, whereby interpretability and robustness are thereby enhanced. This article is motivated by the need for a unified, mathematically sound decision-making framework, through which medical diagnosis under uncertainty can be effectively supported.

**Contributions and novelty:** It is often found that real-world decision-making is complicated by uncertainty, vagueness, and incomplete information, challenges that have long been addressed through fuzzy and dual hesitant sets. The contributions of this article can be summarized as follows.

- A linguistic dual hesitant super hypersoft topological framework is introduced for medical diagnosis, whereby linguistic dual hesitant super hypersoft sets are integrated with topological structures for the first time in this context.
- A linguistic dual hesitant super hypersoft sub-base is employed to generate the topology, through which medical decision problems can be analyzed from both set-theoretic and topological perspectives.
- A distinct yet complementary decision-making algorithm is developed on the basis of linguistic dual hesitant super hypersoft topology, whereby existing fuzzy, intuitionistic fuzzy, and linguistic dual hesitant MCDM approaches are generalized.
- The effectiveness of the proposed framework is validated through numerical examples derived from real-life medical diagnosis scenarios, by which its potential for practical implementation in clinical decision support systems is demonstrated.

**Literature review:** In 1960, Zadeh [13] presented a fuzzy set allows partial membership, with elements assigned values between 0 and 1. Operations like union, intersection and complement extend classical set theory. A separation theorem for convex fuzzy set is established without requiring disjointness, making fuzzy sets useful in decision-making and control systems. Dejian Yu, et al. [6] proposed new aggregation operators for dual hesitant fuzzy sets to better handle uncertainty. Their effectiveness is shown through a numerical example and applied to selecting HR outsourcing suppliers. Baoquan Ning et al. [2] proposed a novel correlation coefficient in the probabilistic dual hesitant fuzzy setting and applies it to a MADM method for evaluating project manager candidates. Sathiyapriya bangarusamy and Dr. V. Pankajam [9] proposed LDHHS maps each medical alternative to a set of linguistic dual hesitant membership-nonmembership pairs defined over a hypersoft multi-attribute parameter space. Bin Zhu and Meimei Xia [5] introduced the concept of Dual Hesitant Fuzzy Sets (DHFSs), explores their properties and operations, and demonstrates their application in group forecasting. Kaviyarasu, et.al [12] proposed a Neutrosophic agricultural topology-based Multi-criteria Decision Making with algorithms validated through real agricultural cases. Maha Mohammed Saeed, et.al [7] introduced eight new concepts in neutrosophic soft bi-topological spaces, focusing on soft open sets and their relation to separation axioms, subspaces, and interiors/closures. Entropy is highlighted as a key structure, illustrated with an engineering example. Baoquan Ning [1] develops novel distance, similarity, and entropy measures for probabilistic dual hesitant fuzzy sets (PDHFS) without altering original membership or non-membership data. Takaaki Fujita, et. al [4] presented Linguistic Indeterm Soft sets and Linguistic Indeterm Hypersoft sets, extending soft set theory to handle layered uncertainty. Shawkat Alkhazaleh, et.al [10] proposed effective fuzzy soft sets to effective fuzzy soft expert sets for handling multi-expert uncertainty. Harish Garg, et.al [3] presented Spherical Fuzzy Soft Topology, defining its fundamental concepts, properties and separation axioms. Suman Das, et.al [11] proposed Neutrosophic Simply Soft Open (NSS-O) sets and NSS-compact sets in neutrosophic soft Topological spaces, studying their properties and related theorems. J. K. Mohan, et.al [8] proposed fuzzy sequence convergence in different fuzzy topologies. New concepts like maximal limit and l-set are introduced with their properties.

In this article, a topological space based on linguistic dual hesitant super hypersoft sets is introduced. Within the methodology, linguistic dual hesitant super hypersoft sets are first constructed, after which open sets are defined within a linguistic dual hesitant super hypersoft topology. Building on this foundation, a new algorithm for medical diagnosis is developed using linguistic dual hesitant super hypersoft topology.

**Acronyms**

LDHFS - Linguistic Dual Hesitant Fuzzy Set

LDHSHS - Linguistic Dual Hesitant Super Hypersoft Set

LDHSHT space - Linguistic Dual Hesitant super hypersoft topological space

LDHSHIT - Linguistic Dual Hesitant Super Hypersoft Indiscrete Topology

LDHSHDT - Linguistic Dual Hesitant Super hypersoft Discrete Topology

LDHSST - Linguistic Dual Hesitant Super soft topology

## 2. Preliminaries

**Definition: 2.1**

Let A = $\{A_1, A_2, A_3, \ldots A_t\}$ where t = 2m + 1: m ≥ 1 and m ∈ $\mathbb{R}^+$ (finite and positive real numbers), be a finite strictly increasing set. For example, if m = 1 then,

A = $\{A_1, A_2, A_3\}$ = {low, medium, high}

For Linguistic set, which is under consideration, the relationship to its elements $A_t$ and the subscript t will be strictly increasing. To define the continuity this set is extended to A = $\{A_\varphi: \varphi \in \mathbb{R}\}$ where φ is also strictly increasing.

**Definition: 2.2**

Let $\underset{\wedge}{U}$ be a fixed set, then a Dual Hesitant Fuzzy Set (DHFS) $\widehat{D}$ on $\underset{\wedge}{U}$ is described as

$\widehat{D} = \{< e, \alpha(e), \beta(e) >| e \in \underset{\wedge}{U}\}$,

in which α(e) and β(e) are two sets of some values in [0, 1], denoting the possible membership degrees and non-membership degrees of the element e ∈ $\underset{\wedge}{U}$ to the set $\widehat{D}$ respectively, with the conditions

$0 \leq \gamma, \eta \leq 1, 0 \leq \gamma^+ + \eta^+ \leq 1$,

Where $\gamma \in \alpha(e)$, $\eta \in \beta(e)$, $\gamma^+ \in \alpha^+(e) = \bigcup_{\gamma\in\alpha} \max\{\gamma\}$ and $\eta^+ \in \beta^+(e) = \bigcup_{\eta\in\beta} \max\{\eta\}$ for all e ∈ $\underset{\wedge}{U}$.

For convenience, the pair $\widehat{D}(e) = (\alpha(e), \beta(e))$ is called a dual hesitant fuzzy element denoted by $\widehat{D}$ = (α, β), with the conditions: $\gamma \in \alpha$, $\eta \in \beta$, $\gamma^+ \in \alpha^+ = \bigcup_{\gamma\in\alpha} \max\{\gamma\}$ and $\eta^+ \in \beta^+ = \bigcup_{\eta\in\beta} \max\{\eta\}$, $0 \leq \gamma, \eta \leq 1$ and $0 \leq \gamma^+ + \eta^+ \leq 1$.

**Definition: 2.3**

Let E = $\{E_1, E_2, E_3, \ldots E_t\}$ for t ≥ 1 be t distinct attributes whose corresponding attributes values are respectively the sets $h_1, h_2, h_3, \ldots h_t$ with $h_x \cap h_y = \emptyset$, for x ≠ y and x, y ∈ {1, 2, 3, . . . t}. Then the pair (F, λ) where λ = $\{h_1 \times h_2 \times h_3 \times \ldots \times h_t$: t is finite and real valued} is known as Hypersoft set over $\breve{\upsilon}$ with mapping

F: $h_1 \times h_2 \times h_3 \times \ldots \times h_t = \lambda \rightarrow$ Ᵽ (U).

**Definition: 2.4**

Let U be a universal set, and let P(U) denote the power set of U. Consider n distinct attributes $a_1, a_2, \ldots a_n$ where n ≥ 1. Each attribute $a_i$ is associated with a set of Attribute values $A_i$, satisfying the property $A_i \cap A_j = \emptyset$ for all i ≠ j. Define $P(A_i)$ as the power set of $A_i$ for each i = 1, 2, . . n. Then, the cartesian product of the power sets of attribute values given by $C = P(A_1) \times P(A_2) \times \ldots \times P(A_n)$. A super hypersoft set over U is a pair (F, C), where $F: C \rightarrow P(U)$, and F maps each element $(\alpha_1, \alpha_2 \ldots\ldots \alpha_n) \in C$ (with $\alpha_i \in P(A_i)$) to a subset $F(\alpha_1, \alpha_2 \ldots\ldots \alpha_n) \subseteq U$. Mathematically, the super hypersoft set is represented as $(F, C) = \{(\gamma, F(\gamma)) \mid \gamma \in C, F(\gamma) \in P(U)\}$. Here, $\gamma = (\alpha_1, \alpha_2 \ldots\ldots \alpha_n) \in C$, where $\alpha_i \in P(A_i)$ for i = 1, 2, . . . , $n$, and $F(\gamma)$ corresponds to the subset of U defined by the combined attribute values $\alpha_1, \alpha_2 \ldots\ldots \alpha_n$.

## 3 Linguistic Dual Hesitant Super Hypersoft Topological Space

**Definition: 3.1**

Let $\ddot{U}$ be a universe of discourse and $\text{Ᵽ}(\ddot{U})$ denote the power set of $\ddot{U}$. Consider t distinct attributes $x_1, x_2, \ldots x_t$ where $t \geq 1$. Each attribute $x_j$ is associated with a set of attribute values $X_j$ satisfying $X_i \cap X_j = \emptyset$ for all $i \neq j$. Let $\text{Ᵽ}(X_j)$ is a power set of $X_j$ for all j = 1, 2, . . . t. Then the cartesian product of the power sets of attribute values is given by $\alpha = \text{Ᵽ}(X_1) \times \ldots\ldots \times \text{Ᵽ}(X_t)$. A Linguistic Dual Hesitant Super Hypersoft Set (LDHSHS) over $\ddot{U}$ is a mapping $A: \alpha \rightarrow \text{Ᵽ}(\ddot{U})$, such that for each $\gamma \in \alpha$, $(A, \alpha) = \{(\gamma, (y, \max(\mu_{A(\gamma)}(y), \nu_{A(\gamma)}(y)) \mid y \in \ddot{U}, \gamma \in \alpha \subseteq \text{Ᵽ}(X_j)\}$
where $\mu_{A(\gamma)}(y)$ and $\nu_{A(\gamma)}(y)$ represent the possible linguistic dual hesitant membership degree and possible linguistic dual hesitant non-membership degree of the element $y \in \ddot{U}$, with the condition $0 \leq \sigma, \zeta \leq 1$, $0 \leq \sigma^+ + \zeta^+ \leq 1$, where $\sigma \in \mu_{A(\gamma)}(y)$, $\zeta \in \nu_{A(\gamma)}(y)$, $\sigma^+ \in \mu_{A(\gamma)}(y) = \bigcup_{\sigma\in\mu} \max\{\sigma\}$ and $\zeta^+ \in \nu_{A(\gamma)}(y) = \bigcup_{\zeta\in\nu} \max\{\zeta\}$ for all $y \in \ddot{U}$. They satisfy the linguistic dual hesitant condition $\mu_{A(\gamma)}(y), \nu_{A(\gamma)}(y) : \ddot{U} \rightarrow [0, 1]$ and $0 \leq \max\{\mu_{G(\underline{E})}(u) + \nu_{G(\underline{E})}(u)\} \leq 1$.
Let LDHSH($\ddot{U}$, α) denote the collection of all linguistic dual hesitant super hypersoft sets over the universe $\ddot{U}$ and $\bar{\tau} \subseteq$ LDHSH($\ddot{U}$, α). Then $\bar{\tau}$ is called Linguistic Dual Hesitant Super Hypersoft Topology (LDHSHT) on $\ddot{U}$ if the following conditions hold

1. $LDHSH_{\phi}(\ddot{U}, \alpha)$, $LDHSH_{\ddot{U}}(\ddot{U}, \alpha)$ belong to $\bar{\tau}$
2. If $(A_1, \alpha)$ and $(A_2, \alpha)$ belong to $\bar{\tau}$ then $(A_1, \alpha) \cap (A_2, \alpha) \in \bar{\tau}$
3. If $\{(A_j, \alpha): j \in J\} \subseteq \bar{\tau}$, then $\bigcup_{j\in J}(A_j, \alpha) \in \bar{\tau}$.

The triplet $(\ddot{U}, \bar{\tau}, \alpha)$ is called a Linguistic Dual Hesitant Super Hypersoft Topological Space (LDHSHT space). Every member of $\bar{\tau}$ is called an open linguistic dual hesitant super hypersoft set. The complement of every open LDHSH($\ddot{U}$, α) is said to be linguistic dual hesitant super hypersoft closed set.

**Definition: 3.2** Let LDHSH($\ddot{U}$, α) denote the collection of all linguistic dual hesitant super hypersoft sets of the universal set $\ddot{U}$.

1. The linguistic dual hesitant super hypersoft topology $\bar{\tau}_{ind} = (LDHSH_{\phi}(\ddot{U}, \alpha))$, $(LDHSH_{\ddot{U}}(\ddot{U}, \alpha))$ is called a Linguistic Dual Hesitant Super Hypersoft Indiscrete Topology (LDHSHIT) on $\ddot{U}$, where $(LDHSH_{\phi}(\ddot{U}, \alpha))$ and $(LDHSH_{\ddot{U}}(\ddot{U}, \alpha))$ denote the empty and whole linguistic dual hesitant super hypersoft sets, respectively. The triplet $(\ddot{U}, \bar{\tau}_{ind}, \gamma)$ is called a linguistic dual hesitant super hypersoft indiscrete topological space.

2. The linguistic dual hesitant super hypersoft topology $\bar{\tau}_{dis} = (LDHSH(\ddot{U}, \text{Ᵽ}(X)))$ is called a Linguistic Dual Hesitant Super Hypersoft Discrete Topology (LDHSHDT) on $\ddot{U}$. The triplet $(\ddot{U}, \bar{\tau}_{dis}, \gamma)$ is called a linguistic dual hesitant super hypersoft discrete topological space.

**Example: 3.1** Let $\ddot{U} = \{y_1, y_2, y_3\}$ be the universe of discourse and $X_1, X_2$ be sets of super attributes. The sets of super attribute values corresponding to $X_1$ and $X_2$ are defined as follows:
$X_1 = \{m_1, m_2\}$, $X_2 = \{n_1, n_2\}$. Clearly, $X_1 \cap X_2 = \emptyset$. Define the power sets of $A_1$ and $A_2$,

$$\text{Ᵽ}(X_1) = \{\emptyset, \{m_1\}, \{m_2\}, \{m_1\ m_2\}\}$$

$Ᵽ(X_2) = \{\emptyset, \{n_1\}, \{n_2\}, \{n_1\ n_2\}\}$.

Thus, the cartesian product of power set of attribute value is $\alpha = Ᵽ(X_1) \times Ᵽ(X_2)$.

Choose four parameters from α: $\gamma_1 = (\{\emptyset\}, \{n_1\ n_2\})$, $\gamma_2 = (\{m_1\}, \{n_1\})$, $\gamma_3 = (\{m_2\}, \emptyset)$ and $\gamma_4 = (\{m_2\}, \{n_1\ n_2\})$. Define A: $\alpha \to Ᵽ(\ddot{U})$, let $\bar{\tau} = \{(LDHSH_{\phi}(\ddot{U}, \alpha)), (LDHSH_{\ddot{U}}(\ddot{U}, \alpha)), (A_1, \alpha), (A_2, \alpha), (A_3, \alpha), (A_4, \alpha)\}$ be a subfamily of $LDHSH(\ddot{U}, \alpha)$, where $(A_1, \alpha)$, $(A_2, \alpha)$, $(A_3, \alpha)$, $(A_4, \alpha)$ are linguistic dual hesitant super hypersoft sets defined as follows

$$(A_1, \alpha) = \{<(\{\emptyset\}, \{n_1\ n_2\}), \{\frac{y_1}{(\text{dissatisfied,medium,satisfied}),(\text{dissatisfied,medium})}, \frac{y_2}{(\text{V.dissatisfied}),(\text{V.satisfied})}, \frac{y_3}{(\text{V.dissatisfied,medium}),(\text{medium})}>\}$$

$$(A_2, \alpha) = \{<(\{m_1\}, \{n_1\}), \{\frac{y_1}{(\text{V.dissatisfied,dissatisfied}),(\text{medium,satisfied})}, \frac{y_2}{(\text{dissatisfied,medium}),(\text{medium,satisfied})}, \frac{y_3}{(\text{dissatisfied,medium}),(\text{medium})}>\}$$

$$(A_3, \alpha) = \{<(\{m_2\}, \emptyset), \{\frac{y_1}{(\text{V.dissatisfied,medium}),(\text{dissatisfied})}, \frac{y_2}{(\text{satisfied}),(\text{dissatisfied,medium})}, \frac{y_3}{(\text{V.dissatisfied}),(\text{medium,satisfied,V.satisfied})}>\}$$

$$(A_4, \alpha) = \{<(\{m_2\}, \{n_1\ n_2\}), \{\frac{y_1}{(\text{dissatisfied,medium}),(\text{medium,satisfied})}, \frac{y_2}{(\text{dissatisfied,medium}),(\text{dissatisfied,medium})}, \frac{y_3}{(\text{V.dissatisfied}),(\text{satisfied,V.satisfied})}>\}.$$

The collection of $\bar{\tau}$ satisfies the conditions of Definition 3.1. Therefore, the triplet $(\ddot{U}, \bar{\tau}, \alpha)$ is a linguistic dual hesitant super hypersoft topological space over the universe $\ddot{U}$.

**Remark 3.1:** It is evident that every linguistic dual hesitant super hypersoft topology induces a Linguistic Dual Hesitant Super Soft Topology (LDHSST). In particular, consider Example 3.1. If the attribute selection is restricted to a single attribute set, say $Ᵽ(X_1)$, then the parameter space reduces to a classical soft parameterization. Consequently, the resulting linguistic dual hesitant super hypersoft topology coincides with a linguistic dual hesitant super soft topology.

Thus, linguistic dual hesitant super hypersoft topology serves as a proper generalization of linguistic dual hesitant super soft topology. Hence, every linguistic dual hesitant super hypersoft topological space reduces to a linguistic dual hesitant super soft topological space when the parameterization is restricted to a single attribute.

However, the converse does not hold in general. Not every linguistic dual hesitant super soft topology can be extended to a linguistic dual hesitant super hypersoft topology. Since super hypersoft topology requires simultaneous multi-attribute parameterization, which may not exist in a given linguistic dual hesitant super soft structure.

**Propositions 3.1:** Let $(\ddot{U}, \bar{\tau}_1, \alpha)$ and $(\ddot{U}, \bar{\tau}_2, \alpha)$ be two linguistic dual hesitant super hypersoft topologies over the universe $\ddot{U}$. Define

$$\bar{\tau}_1 \cap \bar{\tau}_2 = \{(A, \alpha) \mid (A, \alpha) \in \bar{\tau}_1 \text{ and } (A, \alpha) \in \bar{\tau}_2\}$$

Then $\bar{\tau}_1 \cap \bar{\tau}_2$ is also a linguistic dual hesitant super hypersoft topology on $\ddot{U}$.

**Proof:** Since $(\ddot{U}, \bar{\tau}_1, \alpha)$ and $(\ddot{U}, \bar{\tau}_2, \alpha)$ are linguistic dual hesitant super hypersoft topologies over $\ddot{U}$, each topology contains the linguistic dual hesitant super hypersoft empty set $(LDHSH_{\phi}(\ddot{U}, \alpha))$ and the linguistic dual hesitant super hypersoft whole set $(LDHSH_{\ddot{U}}(\ddot{U}, \alpha))$ belong to both $\bar{\tau}_1$ and $\bar{\tau}_2$. Hence $(LDHSH_{\phi}(\ddot{U}, \alpha)), (LDHSH_{\ddot{U}}(\ddot{U}, \alpha)) \in \bar{\tau}_1 \cap \bar{\tau}_2$.

Let $(A_1, \alpha) \cap (A_2, \alpha) \in \bar{\tau}_1 \cap \bar{\tau}_2$. Then $(A_1, \alpha), (A_2, \alpha) \in \bar{\tau}_1$ and $(A_1, \alpha), (A_2, \alpha) \in \bar{\tau}_2$. Since both $\bar{\tau}_1$ and $\bar{\tau}_2$ are linguistic dual hesitant super hypersoft topologies, they are closed under finite intersections. Therefore, $(A_1, \alpha) \cap (A_2, \alpha) \in \bar{\tau}_1$ and $(A_1, \alpha) \cap (A_2, \alpha) \in \bar{\tau}_2$. Thus, $(A_1, \alpha) \cap (A_2, \alpha) \in \bar{\tau}_1 \cap \bar{\tau}_2$.

Let $\{(A_j, \alpha): j \in J\} \subseteq \bar{\tau}_1 \cap \bar{\tau}_2$. Then, for each $j \in J$, $(A_j, \alpha) \in \bar{\tau}_1$ and $(A_j, \alpha) \in \bar{\tau}_2$.
Since both $\hat{\tau}_1$ and $\hat{\tau}_2$ are linguistic dual hesitant super hypersoft topologies, they are closed under arbitrary union. Hence, $\bigcup_{j\in J}(A_j, \alpha) \in \bar{\tau}_1$ and $\bigcup_{j\in J}(A_j, \alpha) \in \bar{\tau}_2$.
Consequently,

$$\bigcup_{j\in J}(A_j, \alpha) \in \bar{\tau}_1 \cap \bar{\tau}_2.$$

**Remark 3.2:** A linguistic dual hesitant super hypersoft topology on a universe $\underset{\cdot\cdot}{U}$ need not be expressible as the union of two linguistic dual hesitant super hypersoft topologies on $\underset{\cdot\cdot}{U}$. In general, the union of two linguistic dual hesitant super hypersoft topologies may fail to satisfy the axioms of a linguistic dual hesitant super hypersoft topology. The following example illustrates this fact.

**Example 3.2** Recall Example 3.1, let $\bar{\tau}_1 = \{(LDHSH_{\phi}(\underset{\cdot\cdot}{U}, \alpha)), (LDHSH_{\underset{\cdot\cdot}{U}}(\underset{\cdot\cdot}{U}, \alpha)), (A_1, \alpha), (A_2, \alpha), (A_3, \alpha)\}$ and $\bar{\tau}_2 = \{((LDHSH_{\phi}(\underset{\cdot\cdot}{U}, \alpha)), (LDHSH_{\underset{\cdot\cdot}{U}}(\underset{\cdot\cdot}{U}, \alpha)), (A_4, \alpha), (A_5, \alpha), (A_6, \alpha), (A_7, \alpha)\}$ where $(A_4, \alpha) = \{<(\{m_1\}, \emptyset), \{\frac{y_1}{(\text{dissatisfied,medium}),(\text{medium,satisfied})}, \frac{y_2}{(\text{V.dissatisfied,dissatisfied}),(\text{medium})}, \frac{y_3}{(\text{V.dissatisfied,dissatisfied}),(\text{medium,satisfied})}>\}$

$(A_5, \alpha) = \{<(\{m_2\}, \{n_1\}), \{\frac{y_1}{(\text{dissatisied}),(\text{medium,satisfied})}, \frac{y_2}{(\text{dissatisfied,medium}),(\text{medium})}, \frac{y_3}{(\text{dissatisfied}),(\text{satisfied})}>\}$

$(A_6, \alpha) = \{<(\{m_1\ m_2\}, \{n_2\}), \{\frac{y_1}{(\text{V.dissatisfied}),(\text{medium,satisfied,V.satisfied})}, \frac{y_2}{(\text{dissatisfied,medium}),(\text{medium,satisfied})}, \frac{y_3}{(\text{V.dissatisfied,satisfied}),(\text{dissatisfied})}>\}$

$(A_7, \alpha) = \{<(\{m_1\ m_2\}, \emptyset), \{\frac{y_1}{(\text{medium}),(\text{medium})}, \frac{y_2}{(\text{V.dissatisfied,satisfied}),(\text{medium})}, \frac{y_3}{(\text{dissatisfied}),(\text{medium,V.satisfied})}>\}$

It is clear that $\bar{\tau}_1 \cap \bar{\tau}_2$ is a linguistic dual hesitant super hypersoft topology. However,

$$(A_3, \alpha) \cup (A_6, \alpha) \notin \bar{\tau}_1 \cup \bar{\tau}_2,$$

which implies that $\bar{\tau}_1 \cup \bar{\tau}_2$ is not a linguistic dual hesitant super hypersoft topology over the universe $\underset{\cdot\cdot}{U}$.

**Proposition 3.2:** Let $(\underset{\cdot\cdot}{U}, \bar{\tau}, \alpha)$ be a linguistic dual hesitant super hypersoft topological space over $\underset{\cdot\cdot}{U}$. Then, for any fixed parameter $\gamma \in \alpha$, the collection

$$\bar{\tau}_{\gamma} = \{A(\gamma): \{(A, \alpha) \in \bar{\tau}\}$$

is a linguistic dual hesitant topology on $\underset{\cdot\cdot}{U}$.

**Proof**

Since $(\underset{\cdot\cdot}{U}, \bar{\tau}, \alpha)$ is a linguistic dual hesitant super hypersoft topological space

$$LDHSH_{\phi}(\underset{\cdot\cdot}{U}, \alpha), LDHSH_{\underset{\cdot\cdot}{U}}(\underset{\cdot\cdot}{U}, \alpha) \in \bar{\tau}.$$

Evaluating these LDHSHS for the fixed element $\gamma \in \alpha$, obtain $LDHSH_{\phi}(\gamma), LDHSH_{\underset{\cdot\cdot}{U}}(\gamma) \in \bar{\tau}_{\gamma}$.
Therefore $\bar{\tau}_{\gamma}$ contains the empty linguistic dual hesitant set and absolute linguistic dual hesitant set.

Let $A_1(\gamma), A_2(\gamma) \in \bar{\tau}_{\gamma}$. Then there exist $(A_1, \alpha), (A_2, \alpha) \in \bar{\tau}$ such that $\gamma \in \alpha$. Since $\bar{\tau}$ is a linguistic dual hesitant super hypersoft topology on $\underset{\cdot\cdot}{U}$, $(A_1, \alpha) \cap (A_2, \alpha) \in \bar{\tau}$. Let $(A_3, \alpha) = (A_1, \alpha) \cap (A_2, \alpha)$. Then $(A_3, \alpha) \in \bar{\tau}$. Note that $A_3(\gamma) = A_1(\gamma) \cap A_2(\gamma)$. Since $(A_3, \alpha) \in \bar{\tau}$, it ollows that $A_3(\gamma) \in \bar{\tau}_{\gamma}$.
Therefore, $A_1(\gamma) \cap A_2(\gamma) \in \bar{\tau}_{\gamma}$. Hence $\bar{\tau}_{\gamma}$ is closed under finite intersections.

Let $\{A_j(\gamma): j \in J\} \subseteq \bar{\tau}_{\gamma}$. Then for every $j \in J$, there exists $(A_j, \alpha) \in \bar{\tau}$ such that $\gamma \in \alpha$. Since $\bar{\tau}$ is a linguistic dual hesitant super hypersoft topology on $\ddot{U}$, $\bigcup_{j\in J}\{(A_j, \alpha): j \in J\} \in \bar{\tau}$. Let $(A, \alpha) = \bigcup_{j\in J}\{(A_j, \alpha): j \in J\}$. Then $(A, \alpha) \in \bar{\tau}$.

Note that $A(\gamma) = \bigcup_{j\in J} A_j(\gamma)$. Since $(A, \alpha) \in \bar{\tau}$, $A(\gamma) \in \bar{\tau}_{\gamma}$. Therefore, $\bigcup_{j\in J} A_j(\gamma) \in \bar{\tau}_{\gamma}$.

Thus, $\bar{\tau}_{\gamma} = \{A(\gamma): (A, \alpha) \in \bar{\tau}\}$ is a linguistic dual hesitant topology on $\ddot{U}$.

**Definition 3.3:** Let $(\ddot{U}, \bar{\tau}, \alpha)$ be a linguistic dual hesitant super hypersoft topological space over $\ddot{U}$ and (A, α) be a linguistic dual hesitant super hypersoft set. The linguistic dual hesitant super hypersoft interior of (A, α) denoted by $\mathrm{int}_{\mathrm{LDHSH}}(A, \alpha)$ is defined as the union of all linguistic dual hesitant super hypersoft open subsets of (A, α). That is, $\mathrm{int}_{\mathrm{LDHSH}}(A, \alpha)$ is the largest linguistic dual hesitant super hypersoft open set contained in (A, α).

**Theorem 3.1:** Let $(\ddot{U}, \bar{\tau}, \alpha)$ be a largest linguistic dual hesitant super hypersoft topological space over $\ddot{U}$ and $(A_1, \alpha), (A_2, \alpha) \in \mathrm{LDHSH}(\ddot{U}, \alpha)$. Then the following properties hold

1. $\mathrm{int}_{\mathrm{LDHSH}}(\mathrm{LDHSH}_{\phi}(\ddot{U}, \alpha)) = (\mathrm{LDHSH}_{\phi}(\ddot{U}, \alpha))$ and $\mathrm{int}_{\mathrm{LDHSH}}(\mathrm{LDHSH}_{\ddot{U}}(\ddot{U}, \alpha)) = (\mathrm{LDHSH}_{\ddot{U}}(\ddot{U}, \alpha))$
2. $\mathrm{int}_{\mathrm{LDHSH}}(A_1, \alpha) \subseteq (A_1, \alpha)$
3. $(A_1, \alpha)$ is a largest linguistic dual hesitant super hypersoft open set iff $\mathrm{int}_{\mathrm{LDHSH}}(A_1, \alpha) = (A_1, \alpha)$
4. $\mathrm{int}_{\mathrm{LDHSH}}(\mathrm{int}_{\mathrm{LDHSH}}(A_1, \alpha) = \mathrm{int}_{\mathrm{LDHSH}}(A_1, \alpha)$
5. If $(A_1, \alpha) \subseteq (A_2, \alpha)$, then $\mathrm{int}_{\mathrm{LDHSH}}(A_1, \alpha) \subseteq \mathrm{int}_{\mathrm{LDHSH}}(A_2, \alpha)$
6. $\mathrm{int}_{\mathrm{LDHSH}}((A_1, \alpha) \cap (A_2, \alpha)) = \mathrm{int}_{\mathrm{LDHSH}}\,(A_1, \alpha) \cap \mathrm{int}_{\mathrm{LDHSH}}(A_2, \alpha)$.

**Proof:** Straight Forward.

**Example 3.3:** Consider the attribute from Example 3.1 Let $\bar{\tau} = \{(\mathrm{LDHSH}_{\phi}(\ddot{U}, \alpha))$, $(\mathrm{LDHSH}_{\ddot{U}}(\ddot{U}, \alpha))$, $(A_1, \alpha)$, $(A_2, \alpha)$, $(A_3, \alpha)$, $(A_4, \alpha)$, $(A_5, \alpha)\}$ is a linguistic dual hesitant super hypersoft topology on $\ddot{U}$. Define

$$(A_5, \alpha) = \{<(\{m_1\ m_2\}, \{n_1\ n_2\}), \{\frac{y_1}{(\text{V.dissatisfied,dissatisfied}),(\text{dissatisfied,satisfied})}, \frac{y_2}{(\text{dissatisfied,medium}),(\text{medium,satisfied})}, \frac{y_3}{(\text{V.dissatisfied}),(\text{medium,V.satisfied})}>\}.$$

Therefore

$\mathrm{int}_{\mathrm{LDHSH}}(A_5, \alpha) = (\mathrm{LDHSH}_{\phi}(\ddot{U}, \alpha)) \cup (A_1, \alpha) \cup (A_2, \alpha) \cup (A_3, \alpha) \cup (A_4, \alpha) = (A_5, \alpha)$.

**Definition 3.4:** Let $(\ddot{U}, \bar{\tau}, \alpha)$ be a linguistic dual hesitant super hypersoft topological space $\ddot{U}$ and (A, α) be a linguistic dual hesitant super hypersoft set. The linguistic dual hesitant super hypersoft closure of (A, α) denoted by $\mathrm{cl}_{\mathrm{LDHSH}}(A, \alpha)$ is defined as the linguistic dual hesitant super hypersoft intersection of all linguistic dual hesitant super hypersoft closed supersets of (A, α). That is, $\mathrm{cl}_{\mathrm{LDHSH}}(A, \alpha)$ is a smallest linguistic dual hesitant super hypersoft closed set is containing (A, α).

**Proposition 3.4:** Let $(\ddot{U}, \bar{\tau}, \alpha)$ be a linguistic dual hesitant super hypersoft topological space over $\ddot{U}$. Then the following properties hold

1. $(\mathrm{LDHSH}_{\phi}(\ddot{U}, \alpha))$ and $(\mathrm{LDHSH}_{\ddot{U}}(\ddot{U}, \alpha))$ are linguistic dual hesitant super hypersoft closet set over $\ddot{U}$.
2. The intersection of any collection of linguistic dual hesitant super hypersoft closed set is a linguistic dual hesitant super hypersoft closed set over $\ddot{U}$.

3. The union of any two linguistic dual hesitant super hypersoft closed set is a linguistic dual hesitant super hypersoft closed set over $\ddot{U}$.

**Proof**

1. Since $(LDHSH_{\phi}(\ddot{U},\alpha)) \in \bar{\tau}$, $(LDHSH_{\phi}(\ddot{U},\alpha))^{c} = (LDHSH_{\phi}(\ddot{U},\alpha))$ is a linguistic dual hesitant super hypersoft closed set, it follows that $(LDHSH_{\ddot{U}}(\ddot{U},\alpha))$ is a LDHSH-closed set. Conversely, if $(LDHSH_{\ddot{U}}(\ddot{U},\alpha)) \in \bar{\tau}$, then $(LDHSH_{\phi}(\ddot{U},\alpha))$ is a linguistic dual hesitant super hypersoft closed set.
2. Let $\{(A_j,\alpha) : j \in J\}$ be a collection of linguistic dual hesitant super hypersoft closed sets, then $\bigcup_{j\in J}(A_j,\alpha)^c \in \bar{\tau}$ . Thus $(\bigcap_{j\in J}(A_j,\alpha))^c \in \bar{\tau}$. Hence, $\bigcap_{j\in J}(A_j,\alpha)$ is a linguistic dual hesitant super hypersoft closed set over $\ddot{U}$.
3. Let $(A_1, \alpha)$, $(A_2, \alpha)$ be two linguistic dual hesitant super hypersoft closed sets. Then $(A_1, \alpha)^c$, $(A_2, \alpha)^c \in \bar{\tau}$. Also, $((A_1, \alpha) \cup (A_2, \alpha))^c = (A_1, \alpha)^c \cap (A_2, \alpha)^c$. Therefore, $((A_1, \alpha) \cup (A_2, \alpha))^c \in \bar{\tau}$ and hence $(A_1, \alpha) \cup (A_2, \alpha)$ is a linguistic dual hesitant super hypersoft closed set.

**Theorem 3.2:** Let $(\ddot{U},\bar{\tau},\alpha)$ be a linguistic dual hesitant super hypersoft topological space over $\ddot{U}$ and, $(A_1, \alpha)$, $(A_2, \alpha) \in LDHSH((\ddot{U},\alpha)$. Then the following properties hold

1. $cl_{LDHSH}(LDHSH_{\phi}(\ddot{U},\alpha)) = LDHSH_{\phi}(\ddot{U},\alpha))$ and $cl_{LDHSH}(LDHSH_{\ddot{U}}(\ddot{U},\alpha)) = (LDHSH_{\ddot{U}}(\ddot{U},\alpha))$
2. $(A_1,\alpha) \subseteq cl_{LDHSH}(A_1,\alpha)$
3. $(A_1,\alpha)$ is a linguistic dual hesitant super hypersoft closed set if and only if $(A_1,\alpha) = cl_{LDHSH}(A_1,\alpha)$
4. $cl_{LDHSH}(cl_{LDHSH}(A_1,\alpha)) = cl_{LDHSH}(A_1,\alpha)$
5. If $(A_1,\alpha) \subseteq (A_2,\alpha)$, then $cl_{LDHSH}(A_1,\alpha) \subseteq cl_{LDHSH}(A_2,\alpha)$
6. $cl_{LDHSH}((A_1,\alpha) \cup (A_2,\alpha)) = cl_{LDHSH}(A_1,\alpha) \cup cl_{LDHSH}(A_2,\alpha)$.

**Proof:** Straight Forward.

**Theorem 3.3:** Let $(\ddot{U},\bar{\tau},\alpha)$ be a linguistic dual hesitant super hypersoft topological space over $\ddot{U}$ and $(A,\alpha) \in LDHSH(\ddot{U},\alpha)$. Then

1. $(cl_{LDHSH}(A,\alpha))^c = int_{LDHSH}((A,\alpha)^c)$,
2. $(int_{LDHSH}(A,\alpha))^c = cl_{LDHSH}((A,\alpha)^c)$.

**Proof**

1. $cl_{LDHSH}(A,\alpha) = \cap\{(A,\alpha): (A,\alpha)$ is closed and $(A,\alpha) \subseteq (A,\alpha)\}$. Taking complements on both sides, $(cl_{LDHSH}(A,\alpha))^c = \cup\{(A,\alpha)^c: (A,\alpha)$ is closed and $(A,\alpha) \subseteq (A,\alpha)\}$. Since the complement of every closed set is open, $(A,\alpha)^c$ is a linguistic dual hesitant super hypersoft open set.

Therefore, $(cl_{LDHSH}(A,\alpha))^c$ is the union of all linguistic dual hesitant super hypersoft open subsets of $(A,\alpha)^c$. By the definition of interior, $(cl_{LDHSH}(A,\alpha))^c = int_{LDHSH}((A,\alpha)^c)$.

2. Similarly, replacing $(A,\alpha)$ by its complement $(A,\alpha)^c$, $(cl_{LDHSH}((A,\alpha))^c)^c = int_{LDHSH}((A,\alpha))$. Taking complements on both sides $(int_{LDHSH}(A,\alpha))^c = cl_{LDHSH}((A,\alpha)^c)$.

**Example 3.4:** Consider the attribute from Example 3.1

Let $\bar{\tau} = \{(LDHSH_{\phi}(\ddot{U},\alpha)), (LDHSH_{\ddot{U}}(\ddot{U},\alpha)), (A_1, \alpha), (A_2, \alpha), (A_3, \alpha), (A_4, \alpha), (A_5, \alpha)\}$ is a linguistic dual hesitant super hypersoft topology on $\ddot{U}$. Define

$$(A_5, \alpha) = \{<(\{m_1\ m_2\}, \{n_1\ n_2\}), \{\frac{y_1}{(V.dissatisfied, dissatisfied), (dissatisfied, satisfied)}, \frac{y_2}{(dissatisfied, medium), (medium, satisfied)}, \frac{y_3}{(V.dissatisfied), (medium, V.satisfied)}>\}.$$

Therefore

$\mathrm{int}_{\mathrm{LDHSH}}(A_5, \alpha) = (\mathrm{LDHSH}_{\phi}(\ddot{U}, \alpha)) \cup (A_1, \alpha) \cup (A_2, \alpha) \cup (A_3, \alpha) \cup (A_4, \alpha) = (A_5, \alpha)$.

**Definition 3.5:** Let $(\ddot{U}, \bar{\tau}, \alpha)$ be a linguistic dual hesitant super hypersoft topological space over $\ddot{U}$ and $\ddot{H} \subseteq \bar{\tau}$. $\ddot{H}$ is called a linguistic dual hesitant super hypersoft basis for the linguistic dual hesitant super hypersoft topology $\bar{\tau}$ if every element of $\bar{\tau}$ can be written as linguistic dual hesitant super hypersoft union of elements of $\bar{\tau}$.

**Proposition 3.4:** Let $(\ddot{U}, \bar{\tau}, \alpha)$ be a linguistic dual hesitant super hypersoft topological space over $\ddot{U}$ and $\ddot{H}$ be a linguistic dual hesitant super hypersoft basis for $\bar{\tau}$. Then $\bar{\tau}$ equals the collection of linguistic dual hesitant super hypersoft unions of elements of $\ddot{H}$.

**Proof**

By the definition of linguistic dual hesitant super hypersoft basis

$$\forall\ (A, \alpha) \in \bar{\tau}, \bigcup_{j \in J} \{(A_j, \alpha)\} \subseteq \ddot{H}$$

such that

$$(A, \alpha) = \bigcup_{j \in J} (A_j, \alpha).$$

That is, for every $y \in \ddot{U}$: $\mu_{A(\gamma)}(y) = \max\ (\max_{j \in J} \mu_{D_j}(y))$, $\nu_{A(\gamma)}(y) = \max\ (\min_{j \in J} \nu_{D_j}(y))$.

Hence, $\bar{\tau} \subseteq \{\bigcup_{j \in J} (A_j, \alpha) : (A_j, \alpha) \in \ddot{H}\}$.

Conversely, each element of $\ddot{H}$ belongs to $\bar{\tau}$, and since $\bar{\tau}$ is closed under arbitrary unions, every union of elements of $\ddot{H}$ also belongs to $\bar{\tau}$

$$\{\bigcup_{j \in J} (A_j, \alpha) : (A_j, \alpha) \in \ddot{H}\} \subseteq \bar{\tau}.$$

Therefore, $\bar{\tau} = \{\bigcup_{j \in J} (A_j, \alpha) : (A_j, \alpha) \in \ddot{H}\}$.

**Example 3.5:** Consider Example 3.1 Then,

$\bar{\tau} = \{(\mathrm{LDHSH}_{\phi}(\ddot{U}, \alpha)), (\mathrm{LDHSH}_{\ddot{U}}(\ddot{U}, \alpha)), (A_1, \alpha), (A_2, \alpha), (A_3, \alpha), (A_5, \alpha)\}$ is a linguistic dual hesitant super hypersoft topology on $\ddot{U}$. Define

$$(A_4, \alpha) = \{<(\{m_1\}, \{n_2\}), \{\frac{y_1}{(\text{dissatisfied}, \text{medium}), (\text{medium})}, \frac{y_2}{(\text{V.dissatisfied}), (\text{medium}, \text{satisfied}, \text{V.satisfied})}, \frac{y_3}{(\text{dissatisfied}, \text{medium}), (\text{medium}, \text{satisfied})}>\}.$$

Therefore,

$$\ddot{H} = \{(\mathrm{LDHSH}_{\phi}(\ddot{U}, \alpha)), (\mathrm{LDHSH}_{\ddot{U}}(\ddot{U}, \alpha)), (A_1, \alpha), (A_2, \alpha), (A_3, \alpha), (A_5, \alpha)\}$$

is a linguistic dual hesitant super hypersoft basis for the linguistic dual hesitant super hypersoft topology $\bar{\tau}$.

**Theorem 3.4:** Let $(\ddot{U}, \bar{\tau}, \alpha)$ be a linguistic dual hesitant super hypersoft topological space over $\ddot{U}$ and $(A, \alpha)$ be a linguistic dual hesitant super hypersoft set over $\ddot{U}$. Then the collection

$$\bar{\tau}_{(A,\alpha)} = \{(A, \alpha) \cap (A_j, \alpha) : (A_j, \alpha) \in \bar{\tau} \text{ for } j \in J\}$$

is a linguistic dual hesitant super hypersoft Topology on the linguistic dual hesitant super hypersoft subset $(A, \alpha)$ relative to the parameter set, super hypersoft parameter family $\alpha$.

**Proof:** $(\mathrm{LDHSH}_{\phi}(\ddot{U}, \alpha)), (\mathrm{LDHSH}_{\ddot{U}}(\ddot{U}, \alpha)) \in \bar{\tau}_{(A,\alpha)}$. Moreover,

$$\bigcap_{j \in J} (A, \alpha)\ \cap\ (A_j, \alpha) = (A, \alpha) \cap (\bigcap_{j \in J} (A_j, \alpha)),$$

and

$$\bigcup_{j\in J}(A,\alpha)\ \cap\ (A_j,\alpha) = (A,\alpha)\cap(\bigcup_{j\in J}(A_j,\alpha)).$$

Thus, the linguistic dual hesitant super hypersoft union of value of linguistic dual hesitant super hypersoft sets in $\bar{\tau}_{(A,\alpha)}$ belongs to $\bar{\tau}_{(A,\alpha)}$ and the finite linguistic dual hesitant super hypersoft intersection of linguistic dual hesitant super hypersoft sets in $\bar{\tau}_{(A,\alpha)}$ belongs to $\bar{\tau}_{(A,\alpha)}$.

Hence, $\bar{\tau}_{(A,\alpha)}$ is a linguistic dual hesitant super hypersoft topology on $(A,\alpha)$.

**Definition 3.6:** Let $(Ṳ,\bar{\tau},\alpha)$ be a linguistic dual hesitant super hypersoft topological space over Ṳ and $(A,\alpha)$ be a linguistic dual hesitant super hypersoft set over Ṳ. Then the linguistic dual hesitant super hypersoft topology

$$\bar{\tau}_{(A,\alpha)} = \{(A,\alpha)\cap(A_j,\alpha): (A_j,\alpha)\in\hat{\tau} \text{ for } j\in J\}$$

is called the linguistic dual hesitant super hypersoft subspace topology and $((A,\alpha),\ \bar{\tau}_{(A,\alpha)},\ \alpha)$ is called a linguistic dual hesitant super hypersoft subspace of $(Ṳ,\bar{\tau},\alpha)$.

**4 LDHSHT space Algorithm to solve MCDM Problem**

Step 1: Consider a universe of Ṳ and a super-attribute set be $\alpha = Ᵽ(X_1)\times\ .\ .\ .\ .\ .\times Ᵽ(X_t)$.

Step 2: Construct the linguistic dual hesitant super hypersoft set $(A,\alpha)$ over Ṳ, corresponding to the super attributes in $\alpha$.

Step 3: Define linguistic dual hesitant super hypersoft topology $\bar{\tau}$, $(A_1, \alpha_1)$, $(A_2, \alpha_2)$ and $(A_3, \alpha_3)$ are open linguistic dual hesitant super hypersoft set in $\bar{\tau}$.

Step 4: Compute the aggregated linguistic dual hesitant super hypersoft sets $(A_1, \alpha_1)$, $(A_2, \alpha_2)$ and $(A_3, \alpha_3)$ in $\bar{\tau}$.

Step 5: Define a linguistic dual hesitant score as

$$\text{Score} = h(\max\{\mu\}) - (\max\{\nu\})$$

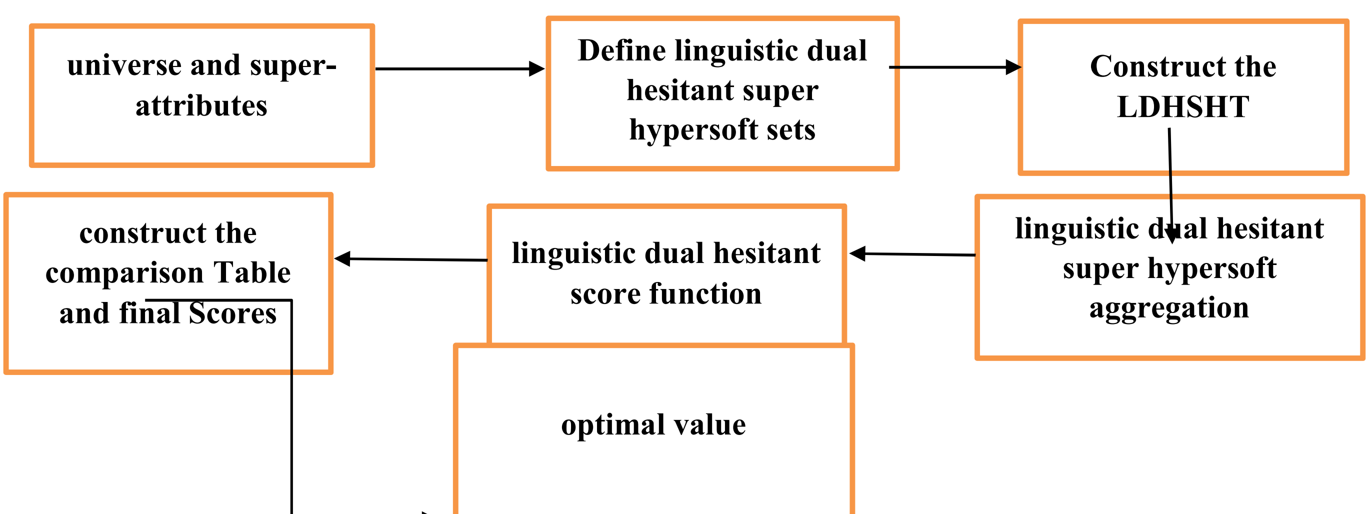
where h is a linguistic term. Let h(extremely dissatisfied) = 0.1, h(dissatisfied) = 0.17, h(medium) = 0.48, h(satisfied) = 0.71 h(extremely satisfied) = 0.92.

Step 6: Construct the comparison table for the resultant linguistic dual hesitant super hypersoft set and compute the row sum $O_j$ and column sum $R_j$ for each alternative $y_j \in Ṳ$. Calculate the score value for each alternative $y_j$ using

$$Z_j = O_j - R_j, \text{ for each element } y_j \in Ṳ.$$

Step 7: Select the optimal alternative $y_j$ such that $y_j = \arg\max\{Z_j\}$. Finally, list the alternatives with maximum membership value. The maximum membership will represent the positive ideal alternative.

**Figure 4.1: Graphical representation of proposed LDHSHT algorithm.**



### 4.1 Illustrative Example

The doctor aims to diagnose five patients, $\ddot{U} = \{y_1, y_2, y_3, y_4, y_5\}$, for the disease acute closure glaucoma. The diagnosis involves uncertainty due to overlapping symptoms, where the symptom attribute set is $X_1 = \{\text{nausea, severe eye pain, seeing halos}\}$, and the diagnostic test attribute set is $X_2 = \{\text{visual acuity test, pupil dilation, pachymetry}\}$. The corresponding super-attribute sets are represented by $Ᵽ(X_1)$ and $Ᵽ(X_2)$. Classical decision-making models ar inadequate for handling the uncertainty and hesitation associated with linguistic membership and non-membership evaluations under multiple super-attributes. Therefore, a linguistic dual hesitant super hypersoft topological space is employed to identify the patient with the highest possibility of being diagnosed with acute closure glaucoma.

**Step: 1**

$\ddot{U} = \{y_1, y_2, y_3, y_4, y_5\}$ be a universal set and a super-attribute set be $\alpha = \{Ᵽ(X_1) \times Ᵽ(X_2)\}$, where $X_1$: symptoms (nausea, severe pain in the eyes, seeing halos) and $X_2$: diagnosis (visual acuity test, pupil dilation, pachymetry).

Let $Ᵽ(X_1)$ = {ϕ, {nausea}, {severe pain in the eyes}, {seeing halos}, {nausea, severe pain in the eyes}, {nausea, seeing halos}, {severe pain in the eyes, seeing halos}, {nausea, severe pain in the eyes, seeing halos}}

$Ᵽ(X_2)$ = {ϕ, {visual acuity test}, {pupil dilation}, {pachymetry}, {visual acuity test, pupil dilation}, {visual acuity test, pachymetry}, {pupil dilation, pachymetry}, {visual acuity test, pupil dilation, pachymetry}}

**Step 2:** Medical experts provide their evaluations in the form of LDHSHSs (($A_1$, α), ($A_2$, α) and ($A_3$, α).

**Step 3:** Construct the Linguistic Dual Hesitant Super Hypersoft Topology (LDHSHT). The goal is to create topology such that these sets are open sets.

$\bar{\tau} = \{(LDHSH_{\phi}(\ddot{U}, \alpha)), (LDHSH_{\ddot{U}}(\ddot{U}, \alpha)), (A_1, \alpha), (A_2, \alpha), (A_3, \alpha)\}$

where ($A_1$, α), ($A_2$, α), ($A_3$, α) are defined as follows

**Table 4.1: Form of ($A_1$, α)**

| **($A_1$, α)** | **$y_1$** | **$y_2$** | **$y_3$** | **$y_4$** | **$y_5$** |
|---|---|---|---|---|---|
| ({nausea, seeing halos}, {visual acuity test, pupil dilation}) = $k_1$ | (medium, dissatisfied), (satisfied, medium) | (dissatisfied, satisfied), (medium) | (satisfied, medium, dissatisfied), (medium, dissatisfied) | (extremely satisfied, medium), (dissatisfied, extremely dissatisfied) | (satisfied, dissatisfied, extremely dissatisfied), (medium, dissatisfied) |
| ({nausea}, {pachymetry}) = $k_2$ | (satisfied, medium, dissatisfied), (extremely dissatisfied) | (satisfied, medium), (medium, dissatisfied) | (extremely satisfied, medium, dissatisfied), (extremely dissatisfied) | (satisfied, medium), (medium, dissatisfied) | (satisfied, medium), (medium) |
| ({severe pain in the eyes, seeing halos}, {visual acuity test, | (satisfied, dissatisfied), (extremely dissatisfied) | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) | (satisfied, dissatisfied), (medium, dissatisfied) | (medium, dissatisfied), (medium, dissatisfied) | (dissatisfied, extremely dissatisfied), (extremely dissatisfied) |

| | | | | | |
|---|---|---|---|---|---|
| pachymetry}) = $k_3$ | | | | | |
| ({seeing halos}, {visual acuity test}) = $k_4$ | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) | (satisfied, medium, dissatisfied), (medium, dissatisfied) | (extremely satisfied, satisfied, medium), (extremely dissatisfied) | (extremely satisfied, medium, dissatisfied), (extremely dissatisfied) | (medium, dissatisfied), (medium, dissatisfied) |
| ({nausea, halos}, {pupil dilation, pachymetry}) = $k_5$ | (satisfied, medium), (dissatisfied, extremely dissatisfied) | (medium, dissatisfied), (medium, dissatisfied) | (extremely satisfied, medium), (extremely dissatisfied) | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) | (satisfied, medium, extremely dissatisfied), (dissatisfied) |

**Table 4.2: Form of ($A_2$, α)**

| **($A_2$, α)** | **$y_1$** | **$y_2$** | **$y_3$** | **$y_4$** | **$y_5$** |
|---|---|---|---|---|---|
| ({nausea, severe pain in the eyes, seeing halos}, {pachymetry}) = $k_6$ | (satisfied, medium, dissatisfied), (extremely dissatisfied) | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) | (satisfied, medium), (dissatisfied, extremely dissatisfied) | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) |
| ({ϕ}, {pupil dilation}) = $k_7$ | (satisfied, medium, dissatisfied), (medium, dissatisfied) | (satisfied, extremely dissatisfied), (medium, dissatisfied) | (extremely satisfied, medium), (extremely dissatisfied) | (medium, dissatisfied), (medium, dissatisfied) | (satisfied, dissatisfied), (medium, dissatisfied) |
| ({nausea, severe pain in the eyes}, {pachymetry}) = $k_8$ | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) | (medium, dissatisfied), (medium, dissatisfied) | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) | (satisfied, medium, dissatisfied), (medium, dissatisfied) | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) |
| ({seeing halos}, {pupil dilation, pachymetry}) = $k_9$ | (dissatisfied, extremely dissatisfied), (extremely dissatisfied) | (satisfied, medium), (dissatisfied, extremely dissatisfied) | (satisfied, dissatisfied), (dissatisfied, extremely dissatisfied) | (medium, dissatisfied), (medium, dissatisfied) | (satisfied, medium, dissatisfied), (medium, dissatisfied) |
| ({severe pain in the eyes, seeing halos}, {visual acuity test, pupil dilation}) = $k_{10}$ | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) | (satisfied, medium), (extremely dissatisfied) | (dissatisfied, extremely dissatisfied), (extremely dissatisfied) | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) | (satisfied, dissatisfied), (medium, dissatisfied) |

**Table 4.3: Form of ($A_3$, α)**

| **($A_3$, α)** | **$y_1$** | **$y_2$** | **$y_3$** | **$y_4$** | **$y_5$** |
|---|---|---|---|---|---|
| ({nausea, severe pain in the | (medium, dissatisfied), | (satisfied, medium, | (medium, extremely | (satisfied, medium), | (medium, dissatisfied), |

| | | | | | |
|---|---|---|---|---|---|
| eyes}, {$\phi$}) = $k_{11}$ | (dissatisfied, extremely dissatisfied) | dissatisfied), (dissatisfied, extremely dissatisfied) | dissatisfied), (medium, extremely dissatisfied) | (dissatisfied, extremely dissatisfied) | (medium, extremely dissatisfied) |
| ({nausea, severe pain in the eyes, seeing halos}, {pupil dilation, pachymetry}) = $k_{12}$ | (medium, dissatisfied), (extremely dissatisfied) | (satisfied, dissatisfied), (medium, extremely dissatisfied) | (satisfied, dissatisfied), (dissatisfied, extremely dissatisfied) | (satisfied, medium), (medium, dissatisfied) | (satisfied, dissatisfied), (medium, dissatisfied, extremely dissatisfied) |
| ({seeing halos}, {visual acuity test, pupil dilation, pachymetry}) = $k_{13}$ | (satisfied, dissatisfied), (medium, dissatisfied) | (satisfied, medium, dissatisfied), (dissatisfied, extremely dissatisfied) | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) |
| ({nausea, halos}, {visual acuity test, pupil dilation, pachymetry}) = $k_{14}$ | (satisfied, medium), (dissatisfied, extremely dissatisfied) | (medium, dissatisfied), (medium, dissatisfied) | (satisfied, medium), (medium, dissatisfied) | (satisfied, dissatisfied), (dissatisfied, extremely dissatisfied) | (medium, dissatisfied), (medium, dissatisfied) |
| ({$\phi$}, {visual acuity test, pupil dilation}) = $k_{15}$ | (satisfied, dissatisfied), (medium, dissatisfied) | (medium, dissatisfied), (dissatisfied, extremely dissatisfied) | (dissatisfied, extremely dissatisfied), (extremely dissatisfied) | (medium, dissatisfied), (medium, dissatisfied) | (satisfied, dissatisfied), (dissatisfied, extremely dissatisfied) |

**Step: 4**

Compute the aggregated linguistic dual hesitant super hypersoft sets ($A_1$, α), ($A_2$, α) and ($A_3$, α) in $\overline{\tau}$.

**Table 4.4: Comparison table of ($A_1$, α) ∨ ($A_2$, α) ∨ ($A_3$, α)**

| **($A_1$, α) ∨ ($A_2$, α) ∨ ($A_3$, α)** | **$y_1$** | **$y_2$** | **$y_3$** | **$y_4$** | **$y_5$** |
|---|---|---|---|---|---|
| $k_1 \times k_6 \times k_{11}$ | (satisfied, extremely dissatisfied) | (satisfied, dissatisfied) | (satisfied, dissatisfied) | (satisfied, extremely dissatisfied) | (satisfied, dissatisfied) |
| $k_2 \times k_7 \times k_{12}$ | (satisfied, extremely dissatisfied) | (satisfied, extremely dissatisfied) | (extremely satisfied, extremely dissatisfied) | (satisfied, dissatisfied) | (satisfied, dissatisfied) |
| $k_3 \times k_8 \times k_{13}$ | (satisfied, extremely dissatisfied) | (satisfied, dissatisfied) | (satisfied, dissatisfied) | (satisfied, dissatisfied) | (medium, extremely dissatisfied) |
| $k_4 \times k_9 \times k_{14}$ | (satisfied, extremely dissatisfied) | (satisfied, dissatisfied) | (extremely satisfied, | (extremely satisfied, | (satisfied, medium) |

| | | | | | |
|---|---|---|---|---|---|
| | | | extremely dissatisfied) | extremely dissatisfied) | |
| $k_5 \times k_{10} \times k_{15}$ | (satisfied, dissatisfied) | (satisfied, extremely dissatisfied) | (extremely satisfied, extremely dissatisfied) | (medium, dissatisfied) | (satisfied, dissatisfied) |

**Step: 5**

Define a linguistic dual hesitant score as

$$\text{Score} = h(\max\{\mu\}) - (\max\{\nu\})$$

where h is a linguistic term. Let h(extremely dissatisfied) = 0.1, h(dissatisfied) = 0.17, h(medium) = 0.48, h(satisfied) = 0.71 h(extremely satisfied) = 0.92.

**Step: 6**

**Table 4.5: Score value**

| $(A_1, \alpha) \vee (A_2, \alpha) \vee (A_3, \alpha)$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | Row ($D_j$) |
|---|---|---|---|---|---|---|
| $k_1 \times k_6 \times k_{11}$ | 0.61 | 0.54 | 0.54 | 0.61 | 0.54 | 2.84 |
| $k_2 \times k_7 \times k_{12}$ | 0.61 | 0.61 | 0.82 | 0.54 | 0.54 | 3.12 |
| $k_3 \times k_8 \times k_{13}$ | 0.61 | 0.54 | 0.54 | 0.54 | 0.38 | 2.61 |
| $k_4 \times k_9 \times k_{14}$ | 0.61 | 0.54 | 0.82 | 0.82 | 0.23 | 3.02 |
| $k_5 \times k_{10} \times k_{15}$ | 0.54 | 0.61 | 0.82 | 0.31 | 0.54 | 2.82 |
| **Column ($C_j$)** | 2.98 | 2.84 | 3.54 | 2.82 | 2.23 | |

The score $Z_j$ for each $y_j$ (j = 1, 2, 3, 4, 5) is computed as: $Z_j = O_j - R_j$.

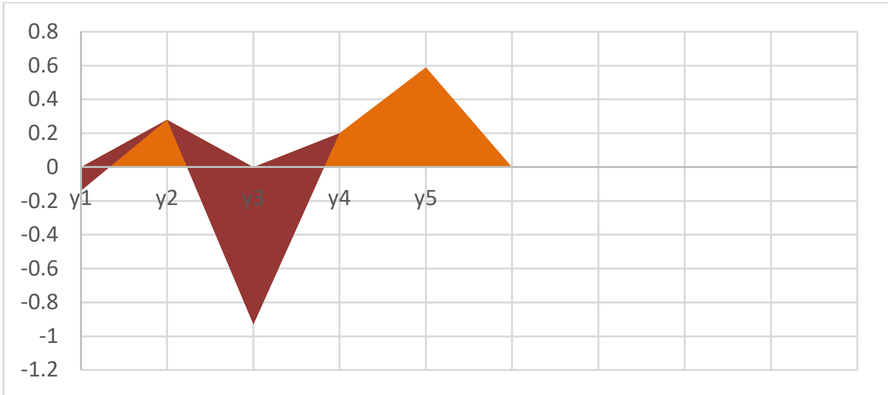
$$\begin{array}{ccc} \textbf{Patients} & & \textbf{Score} \\ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} & = & \begin{bmatrix} -0.14 \\ 0.28 \\ -0.93 \\ 0.2 \\ 0.59 \end{bmatrix} \end{array}$$

**Step:7**

The optimal alternative $y_j = \arg\max \{Z_j\}$ = row ($O_j$) – column ($R_j$), patient $y_5$, $y_4$, $y_2$ is most likely to have acute closure glaucoma because it has the positive score values than other values of $y_1$ and $y_3$.

$$\begin{array}{ccccc} \textbf{Patients} & & \textbf{Score} & & \textbf{Result} \\ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} & = & \begin{bmatrix} -0.14 \\ 0.28 \\ -0.93 \\ 0.2 \\ 0.59 \end{bmatrix} & = & \begin{bmatrix} \text{negative} \\ \text{positive} \\ \text{negative} \\ \text{positive} \\ \text{positive} \end{bmatrix} \end{array}$$

**Figure 7.2.2:** Negative and Positive Patients in Acute closure Glaucoma




represent positive in Acute closure Glaucoma
represent negative in Acute closure Glaucoma

**Result:** Let Ṳ = {$y_1$, $y_2$, $y_3$, $y_4$, $y_5$} denote the set of possible diagnostic alternatives, and let (Ṳ, $\overline{\tau}$, $\gamma$) be a linguistic dual hesitant super hypersoft topological space generated by a linguistic dual hesitant super hypersoft sub-base. Suppose that ($A_1$, $\alpha$), ($A_2$, $\alpha$) and ($A_3$, $\alpha$) are linguistic dual hesitant super hypersoft sets representing evaluations provided by medical experts, with all sets belonging to $\overline{\tau}$. By employing the aggregation operator $\vee$, a combined linguistic dual hesitant hypersoft set (A, $\alpha$) is obtained, which remains open within the topology $\overline{\tau}$. For each alternative $y_j \in$ Ṳ, evaluation measures $O_j$ and $R_j$ are computed through a suitable neutrosophic score function $Z_j$, leading to an overall diagnostic score $Z_j = O_j - R_j$. The numerical outcomes indicate that the alternative $y_j$, satisfying $y_j = \arg\max \{Z_j\}$, is consistently identified as the most appropriate diagnostic or treatment option, thereby validating the effectiveness of the proposed framework in medical decision-making under uncertainty. The results revealed that patient $y_5$, $y_4$, $y_2$ is most likely to have acute closure glaucoma because it has the positive score values than other values of $y_1$, $y_3$.

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