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LINGUISTIC PYTHAGOREAN SUPER HYPERSOFT TOPOLOGICAL SPACE IN MULTI CRITERIA DECISION MAKING

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Abstract

In this article topological framework of linguistic pythagorean super hypersoft set for medical diagnosis in uncertain multi-attribute settings. It uses pythagorean values and topological concepts such as subbase, interior, and closure. An algorithm integrating linguistic pythagorean super hypersoft set evaluation with topological operations such as interior and closure is developed for effective ranking and decision-making. A score- based ranking method is used to select optimal alternatives. The linguistic pythagorean super hypersoft topology method improves decision accuracy and reliability in medical diagnosis.

Keywords: Linguistic Pythagorean set, hypersoft set, super hypersoft set, super hypersoft topology, multi criteria decision making.

1. Introduction: fuzzy sets have been introduced to handle different levels of uncertainty by incorporating degrees of membership and non-membership. Linguistic formulations possess inherent qualities of ambiguity, imprecision, and vagueness. However, these models face limitations when dealing with hierarchical parameter structures and super hypersoft attribute family decision problems.

The concept of super hypersoft sets was proposed as a generalization of hypersoft sets by extending super hypersoft attribute family functions. When combined with linguistic pythagorean, linguistic pythagorean super hypersoft sets provide a flexible framework for modelling membership and non-membership across super hypersoft attribute families. Furthermore, the introduction of linguistic pythagorean super hypersoft topology enhances the structural analysis of such sets by defining open sets and enabling topological operations within a linguistic pythagorean super hypersoft set environment.

Motivation and research gap: motivation and research gap for this work arise from the growing complexity of medical diagnostic processes, where decisions must be made based on heterogeneous, uncertain, and sometimes contradictory information. In many real-world clinical situations, symptoms may not clearly indicate a disease, test results may be inconclusive, and the super hypersoft attribute family may differ.

linguistic pythagorean super hypersoft sets provide a natural mathematical structure to address these challenges by allowing independent modeling of membership and non-membership across combinations of medical attributes. However, most existing linguistic Pythagorean-based decision models lack a topological perspective, which can offer valuable insights into the structural relationships among diagnostic alternatives. Introducing a linguistic Pythagorean super hypersoft topology enables the analysis of medical decisions through open sets and topological operators, thereby enhancing interpretability and robustness. This article is motivated by the need for a unified, mathematically sound decision-making framework that can effectively support medical diagnosis under uncertainty.

Contributions and novelty: In this article decision-making in real-world environments often involves uncertainty, vagueness, and incomplete information. Fuzzy sets and Pythagorean. This article can be summarized as follows. First, a linguistic pythagorean super hypersoft topological framework is introduced for medical diagnosis, which integrates linguistic pythagorean super hypersoft sets with topological structures for the first time in this context. Second, a linguistic pythagorean super hypersoft sub-base is employed to generate the topology, enabling a pythagorean analysis of medical decision problems through both set-theoretic and topological perspectives. Third, a distinct yet complementary decision-making algorithm is developed based on linguistic pythagorean super hypersoft topology, thereby generalizing existing fuzzy, intuitionistic fuzzy, and linguistic pythagorean MCDM approaches. Finally, the effectiveness of the proposed framework is validated through numerical examples derived from real-life medical diagnosis scenarios, demonstrating its potential for practical implementation in clinical decision support systems.

Literature review: In 1960, Zadeh [9] presented a fuzzy set allows partial membership, with elements assigned values between 0 and 1. Operations like union, intersection and complement extend classical set theory. A separation theorem for convex fuzzy set is established without requiring disjointness, making fuzzy sets useful in decision-making and control systems. Kaviyarasu, et.al [7] proposed a Neutrosophic agricultural topology-based Multi-criteria Decision Making with algorithms validated through real agricultural cases. Harish Garg [2] proposed Linguistic Pythagorean fuzzy sets for decision-making under uncertainty and validates them with examples. Takaaki Fujita, et. al [3] presented Linguistic Indeterm Soft sets and Linguistic Indeterm Hypersoft sets, extending soft set theory to handle layered uncertainty. Sathiyapriya Bangarusamy, et.al [5] proposed LPHS enhances diagnostic accuracy, supports tailored strategies and improves decision-making in healthcare. Takaaki Fujita, et.al [8] explored the quantum hypersoft Set and quantum super hypersoft Set by merging hypersoft and superhypersoft structures with quantum superposition, enabling probabilistic membership for future decision-analysis, topological, and algebraic applications. Muhammad Akram, et.al [1] explored a complex neutrosophic soft topology and use it with Cotangent Similarity to solve signal-template alignment, finding S2–T1 (CotSM = 0.6653) as the strongest match and S2 as the most significant signal overall. Zanyar A. Ameen, et.al [10] presented new weak classes of soft open sets, focusing on the Baire property. Muhammad Harish Garg, et.al [4] presented Spherical Fuzzy Soft Topology, defining its fundamental concepts, properties and separation axioms. Suman Das, et.al [6] proposed Neutrosophic Simply Soft Open (NSS-O) sets and NSS-compact sets in neutrosophic soft Topological spaces, studying their properties and related theorems.

In this article, the linguistic pythagorean super hypersoft sets topological space is introduced. The methodology begins with the construction of linguistic pythagorean soft expert sets, followed by the definition of open sets within a linguistic pythagorean soft expert topology. A new algorithm for medical diagnosis is developed using linguistic pythagorean soft expert topology.

Acronyms

LPFV - Linguistic Pythagorean Fuzzy value

LPFS - Linguistic Pythagorean Fuzzy Set

LPFN - Linguistic Pythagorean Fuzzy Number

LPSHS - Linguistic Pythagorean Super Hypersoft Set

LPSHT space - linguistic pythagorean super hypersoft topological space

LPSHIT - Linguistic Pythagorean Super Hypersoft Indiscrete Topology
LPSHDT - Linguistic Pythagorean Super hypersoft Discrete Topology
LPSST - Linguistic Pythagorean Super soft topology

2. Preliminaries

Definition 2.1 (Linguistic Pythagorean Set)

Let X be a universal set and B = $\{b_\varphi: b_0 \le b_\varphi \le b_t,\ \varphi \in [0, t]\}$ be a continuous linguistic term set. A Linguistic Pythagorean Fuzzy Set (LPFS) is defined in the finite universe of discourse X mathematically with the form

$$Ł = \{(a, b_\alpha(a), b_\beta(a)) \mid a \in X\}$$

where $b_\alpha(a)$, $b_\beta(a) \in$ B stand for the linguistic membership degree and linguistic non-membership degree of the element a to Ł. We shall denote the pairs of $(b_\alpha(a), b_\beta(a))$ as Ł $= (b_\alpha, b_\beta)$ and called as linguistic Pythagorean fuzzy value (LPFV) or linguistic Pythagorean fuzzy number (LPFN).

Definition 2.2 (Hypersoft Set)

Let E = $\{E_1, E_2, E_3, \ldots E_t\}$ for $t \ge 1$ be t distinct attributes whose corresponding attributes values are respectively the sets $h_1, h_2, h_3, \ldots h_t$ with $h_x \cap h_y = \emptyset$, for $x \ne y$ and x, y ∈ {1, 2, 3, . . . t}. Then the pair (F, λ) where $\lambda = \{h_1 \times h_2 \times h_3 \times \ldots \times h_t$: t is finite and real valued} is known as Hypersoft set over U with mapping

F: $h_1 \times h_2 \times h_3 \times \ldots \times h_t = \lambda \rightarrow$ Ᵽ (U).

Definition 2.3 (Super Hypersoft Set)

Let U be a universal set, and let P(U) denote the power set of U. Consider n distinct attributes $a_1, a_2, \ldots a_n$ where $n \ge 1$. Each attribute a_i is associated with a set of Attribute values A_i, satisfying the property $A_i \cap A_j = \emptyset$ for all $i \ne j$. Define $P(A_i)$ as the power set of A_i for each i = 1, 2, . . n. Then, the cartesian product of the power sets of attribute values given by $C = P(A_1) \times P(A_2) \times \ldots \times P(A_n)$. A super hypersoft set over U is a pair (F, C), where F: C → P(U), and F maps each element $(\alpha_1, \alpha_2 \ldots\ldots \alpha_n) \in C$ (with $\alpha_i \in P(A_i)$) to a subset $F(\alpha_1, \alpha_2 \ldots\ldots \alpha_n) \subseteq U$. Mathematically, the super hypersoft set is represented as $(F, C) = \{(\gamma, F(\gamma)) \mid \gamma \in C, F(\gamma) \in P(U)\}$. Here, $\gamma = (\alpha_1, \alpha_2 \ldots\ldots \alpha_n) \in C$, where $\alpha_i \in P(A_i)$ for $i = 1, 2, \ldots, n$, and $F(\gamma)$ corresponds to the subset of U defined by the combined attribute values $\alpha_1, \alpha_2 \ldots\ldots \alpha_n$.

3. Linguistic Pythagorean Super Hypersoft Topological Space

Definition 3.1

Let Ụ be a universe of discourse and Ᵽ(Ụ) denote the power set of Ụ. Consider n distinct attributes $a_1, a_2, \ldots a_n$ where $n \ge 1$. Each attribute a_i is associated with a set of attribute values A_i satisfying $A_i \cap A_j = \emptyset$ for all $i \ne j$. Let Ᵽ(A_i) denote the power set of A_i for all i = 1, 2, . . . n. Then the cartesian product of the power sets of the attribute values is given by λ = Ᵽ(A_1) × . . . . . × Ᵽ(A_n). A Linguistic Pythagorean Super Hypersoft Set (LPSHS) over Ụ is a mapping G: λ → Ᵽ(Ụ), such that for each δ ∈ λ,

(G, λ) = {(δ, (u, $\mu_{G(\delta)}(u)$, $\nu_{G(\delta)}(u)$) | u ∈ Ụ, δ ∈ λ ⊆ Ᵽ(A_i)}

where $\mu_{G(\delta)}(u)$ and $\nu_{G(\delta)}(u)$ represent the linguistic pythagorean membership and linguistic pythagorean non-membership respectively. The linguistic pythagorean membership and non-membership functions of an LPSHS are defined as $\mu_{G(\delta)}(u), \nu_{G(\delta)}(u)$: Ụ → [0, 1] satisfying $0 \le (\mu_{G(\delta)}(u))^2 + (\nu_{G(\delta)}(u))^2 \le 1$.

Let LPSH($\underset{\wedge}{U}$, λ) denote the family of all linguistic pythagorean super hypersoft sets over the universe $\underset{\wedge}{U}$ and $\hat{\tau} \subseteq$ LPSH($\underset{\wedge}{U}$, λ). Then $\hat{\tau}$ is called linguistic pythagorean super hypersoft topology on $\underset{\wedge}{U}$ if the following conditions hold

1. $\mathrm{LPSH}_{\phi}(\underset{\wedge}{U}, \lambda)$, $\mathrm{LPSH}_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)$ belong to $\hat{\tau}$
2. If (G_1, λ) and (G_2, λ) belong to $\hat{\tau}$ then $(G_1, \lambda) \cap (G_2, \lambda) \in \hat{\tau}$
3. If $\{(G_i, \lambda)\colon i \in I\} \subseteq \hat{\tau}$, then $\bigcup_{i \in I}(G_i, \lambda) \in \hat{\tau}$.

The triplet $(\underset{\wedge}{U}, \hat{\tau}, \lambda)$ is called a linguistic pythagorean super hypersoft topological space (LPSHT space). Every member of $\hat{\tau}$ is called an open linguistic pythagorean super hypersoft set. The complement of every open LPSHS is called a linguistic pythagorean super hypersoft closed set.

Definition 3.2: Let LPSH($\underset{\wedge}{U}$, λ) denote the family of all linguistic pythagorean super hypersoft sets over the universe $\underset{\wedge}{U}$.

1. The linguistic pythagorean super hypersoft topology $\hat{\tau}_{ind} = \{\mathrm{LPSH}_{\phi}(\underset{\wedge}{U}, \lambda), \mathrm{LPSH}_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)\}$ is called a Linguistic Pythagorean Super Hypersoft Indiscrete Topology (LPSHIT) on $\underset{\wedge}{U}$, where $(\mathrm{LPSH}_{\phi}(\underset{\wedge}{U}, \lambda))$ and $(\mathrm{LPSH}_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda))$ denote the null and absolute linguistic pythagorean super hypersoft Sets, respectively. The triplet ($\underset{\wedge}{U}$, $\hat{\tau}_{ind}$, λ) is called a linguistic pythagorean super hypersoft indiscrete topological space.

2. The linguistic pythagorean super hypersoft topology $\hat{\tau}_{dis}$ = LPSH($\underset{\wedge}{U}$, λ) is called a Linguistic Pythagorean Super hypersoft Discrete Topology (LPSHDT) on $\underset{\wedge}{U}$. The triplet ($\underset{\wedge}{U}$, $\hat{\tau}_{dis}$, λ) is called a linguistic pythagorean super hypersoft discrete topological space.

Example 3.3. Let $\underset{\wedge}{U} = \{u_1, u_2, u_3\}$ be the universe of discourse and A_1, A_2 be sets of super attributes. The sets of super attribute values corresponding to A_1 and A_2 are defined as follows:

$A_1 = \{s_1, s_2\}$, $A_2 = \{t_1, t_2\}$. Clearly, $A_1 \cap A_2 = \emptyset$. Define the power sets of A_1 and A_2,

$$\mathbb{P}(A_1) = \{\emptyset, \{s_1\}, \{s_2\}, \{s_1, s_2\}\}$$
$$\mathbb{P}(A_2) = \{\emptyset, \{t_1\}, \{t_2\}, \{t_1, t_2\}\}.$$

Thus, the Cartesian Product of the power sets of attribute value is $\lambda = \mathbb{P}(A_1) \times \mathbb{P}(A_2)$.

Choose four parameters from λ: $\delta_1 = (\{s_1\}, \{t_2\})$, $\delta_2 = (\{s_2\}, \{t_1\ t_2\})$, $\delta_3 = (\{s_2\}, \{t_1\})$ and $\delta_4 = (\{s_1\ s_2\}, \{t_2\})$. Define G: $\lambda \to \mathbb{P}(\underset{\wedge}{U})$, let $\hat{\tau} = \{(\mathrm{LPSH}_{\phi}(\underset{\wedge}{U}, \lambda)), (\mathrm{LPSH}_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)), (G_1, \lambda), (G_2, \lambda), (G_3, \lambda), (G_4, \lambda)\}$ be a subfamily of LPSH($\underset{\wedge}{U}$, λ), where (G_1, λ), (G_2, λ), (G_3, λ), (G_4, λ) are linguistic pythagorean super hypersoft sets defined as follows:

$$(G_1, \lambda) = \{<(\{s_1\}, \{t_2\}), \{\frac{u_1}{(\text{satisfied,dissatisfied})}, \frac{u_2}{(\text{V.dissatisfied,satisfied})}, \frac{u_3}{(\text{dissatisfied,medium})}>\}$$

$$(G_2, \lambda) = \{<(\{s_2\}, \{t_1\ t_2\}), \{\frac{u_1}{(\text{medium,medium})}, \frac{u_2}{(\text{medium,satisfied})}, \frac{u_3}{(\text{dissatisfied,satisfied})}>\}$$

$$(G_3, \lambda) = \{<(\{s_2\}, \{t_1\}), \{\frac{u_1}{(\text{dissatisfied,satisfied})}, \frac{u_2}{(\text{satisfied,medium})}, \frac{u_3}{(\text{satisfied,medium})}>\}$$

$$(G_4, \lambda) = \{<(\{s_1\}, \{t_1\}), \{\frac{u_1}{(\text{medium,satisfied})}, \frac{u_2}{(\text{dissatisfied,satisfied})}, \frac{u_3}{(\text{V.dissatisfied,satisfied})}>\}.$$

The collection $\hat{\tau}$ satisfies the conditions of Definition 3.1. Therefore, the triplet $(\underset{\wedge}{U}, \hat{\tau}, \lambda)$ is a linguistic pythagorean super hypersoft topological space over the universe $\underset{\wedge}{U}$.

Remark 3.1 It is evident that every linguistic pythagorean super hypersoft topology induces a Linguistic Pythagorean Super soft topology (LPSST). In particular, consider Example 3.1. If the parameter selection is restricted to a single parameter set, say $\mathbb{P}(A_1)$, then the parameter space reduces to a classical soft parameterization. Consequently, the resulting linguistic pythagorean super hypersoft topology coincides with a linguistic pythagorean super soft topology.

Thus, linguistic pythagorean super hypersoft topology serves as a proper generalization of linguistic pythagorean super soft topology. Hence, every linguistic pythagorean super hypersoft topological Space reduces to a Linguistic Pythagorean Super Soft Topological Space when the parameterization is restricted to a single attribute.

However, the converse does not hold in general. Not every linguistic pythagorean super soft topology can be extended to a linguistic pythagorean super hypersoft topology, since super hypersoft topology requires simultaneous multi-attribute parameterization, which may not exist in a given linguistic pythagorean super soft structure.

Propositions 3.1: Let $(\underset{\wedge}{U}, \hat{\tau}_1, \lambda)$ and $(\underset{\wedge}{U}, \hat{\tau}_2, \lambda)$ be two linguistic pythagorean super hypersoft topologies over the universe $\underset{\wedge}{U}$. Define

$$\hat{\tau}_1 \cap \hat{\tau}_2 = \{(G, \lambda) \mid (G, \lambda) \in \hat{\tau}_1 \text{ and } (G, \lambda) \in \hat{\tau}_2\}$$

Then $\hat{\tau}_1 \cap \hat{\tau}_2$ is also a linguistic pythagorean super hypersoft topology on $\underset{\wedge}{U}$.

Proof: Since $(\underset{\wedge}{U}, \hat{\tau}_1, \lambda)$ and $(\underset{\wedge}{U}, \hat{\tau}_2, \lambda)$ are linguistic pythagorean super hypersoft topologies over $\underset{\wedge}{U}$, each topology contains the linguistic pythagorean super hypersoft null set $(LPSH_{\phi}(\underset{\wedge}{U}, \lambda))$ and the linguistic pythagorean super hypersoft absolute set $(LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda))$ belong to both $\hat{\tau}_1$ and $\hat{\tau}_2$. Hence $(LPSH_{\phi}(\underset{\wedge}{U}, \lambda))$, $(LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)) \in \hat{\tau}_1 \cap \hat{\tau}_2$.

Let $(G_1, \lambda), (G_2, \lambda) \in \hat{\tau}_1 \cap \hat{\tau}_2$. Then $(G_1, \lambda), (G_2, \lambda) \in \hat{\tau}_1$ and $(G_1, \lambda), (G_2, \lambda) \in \hat{\tau}_2$. Since both $\hat{\tau}_1$ and $\hat{\tau}_2$ are linguistic pythagorean super hypersoft topologies, they are closed under finite intersections. Therefore, $(G_1, \lambda) \cap (G_2, \lambda) \in \hat{\tau}_1$ and $(G_1, \lambda) \cap (G_2, \lambda) \in \hat{\tau}_2$. Thus, $(G_1, \lambda) \cap (G_2, \lambda) \in \hat{\tau}_1 \cap \hat{\tau}_2$.

Let $\{(G_i, \lambda): i \in I\} \subseteq \hat{\tau}_1 \cap \hat{\tau}_2$. Then, for each $i \in I$, $(G_i, \lambda) \in \hat{\tau}_1$ and $(G_i, \lambda) \in \hat{\tau}_2$.

Since both $\hat{\tau}_1$ and $\hat{\tau}_2$ are linguistic pythagorean super hypersoft topologies, they are closed under arbitrary union. Hence, $\bigcup_{i \in I}(G_i, \lambda) \in \hat{\tau}_1$ and $\bigcup_{i \in I}(G_i, \lambda) \in \hat{\tau}_2$.

Consequently,

$$\bigcup_{i \in I}(G_i, \lambda) \in \hat{\tau}_1 \cap \hat{\tau}_2.$$

Remark 3.2: A linguistic pythagorean super hypersoft topology on a universe $\underset{\wedge}{U}$ need not be expressible as the union of two linguistic pythagorean super hypersoft topologies on $\underset{\wedge}{U}$. In general, the union of two linguistic pythagorean super hypersoft topologies may fail to satisfy the axioms of a linguistic pythagorean super hypersoft topology. The following example illustrates this fact.

Example 3.2 Recall Example 3.1. Let $\hat{\tau}_1 = \{(LPSH_{\phi}(\underset{\wedge}{U}, \lambda)), (LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)), (G_1, \lambda), (G_2, \lambda), (G_3, \lambda), (G_4, \lambda)\}$ and $\hat{\tau}_2 = \{(LPSH_{\phi}(\underset{\wedge}{U}, \lambda)), (LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)), (G_5, \lambda), (G_6, \lambda), (G_7, \lambda), (G_8, \lambda), (G_9, \lambda)\}$, where

$$(G_5, \lambda) = \{<(\phi, \{t_2\}), \{\frac{u_1}{(\text{dissatisfied,medium})}, \frac{u_2}{(\text{V.dissatisfied,medium})}, \frac{u_3}{(\text{dissatisfied,satisfied})}>\}$$

$$(G_6, \lambda) = \{<(\{s_1\}, \{t_2\}), \{\frac{u_1}{(\text{V.dissatisied,medium})}, \frac{u_2}{(\text{medium,satisfied})}, \frac{u_3}{(\text{V.dissatisfied,satisfied})}>\}$$

$$(G_7, \lambda) = \{<(\{s_1\}, \{t_1, t_2\}), \{\frac{u_1}{(\text{dissatisfied,medium})}, \frac{u_2}{(\text{dissatisfied,satisfied})}, \frac{u_3}{(\text{medium,medium})}>\}$$

$(G_8, \lambda) = \{<(\{s_1\}, \phi), \{\frac{u_1}{(\text{medium,satisfied})}, \frac{u_2}{(\text{V.dissatisfied,satisfied})}, \frac{u_3}{(\text{V.dissatisfied,V.satisfied})}>\}$

$(G_9, \lambda) = \{<(\{s_1, s_2\}, \{t_1, t_2\}), \{\frac{u_1}{(\text{medium,medium})}, \frac{u_2}{(\text{V.dissatisfied,V.satisfied})}, \frac{u_3}{(\text{V.dissatisfied,medium})}>\}.$

It is clear that $\hat{\tau}_1 \cap \hat{\tau}_2$ is a linguistic pythagorean super hypersoft topology. However,

$$(G_1, \lambda) \cup (G_5, \lambda) \notin \hat{\tau}_1 \cup \hat{\tau}_2,$$

which implies that $\hat{\tau}_1 \cup \hat{\tau}_2$ is not a linguistic pythagorean super hypersoft topology over the universe $\underset{\wedge}{U}$.

Proposition 3.2: Let $(\underset{\wedge}{U}, \hat{\tau}, \lambda)$ be a linguistic pythagorean super hypersoft topological space over $\underset{\wedge}{U}$. Then, for any fixed parameter $\delta \in \lambda$, the collection

$$\hat{\tau}_\delta = \{G(\delta):(G, \lambda) \in \hat{\tau}\}$$

is a linguistic pythagorean topology on $\underset{\wedge}{U}$.

Proof

Since $(\underset{\wedge}{U}, \hat{\tau}, \lambda)$ is a linguistic pythagorean super hypersoft topological space

$$LPSH_\phi(\underset{\wedge}{U}, \lambda),\ LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda) \in \hat{\tau}.$$

Evaluating these LPSHS for the fixed element $\delta \in \lambda$, obtain $LPSH_\phi(\delta)$, $LPSH_{\underset{\wedge}{U}}(\delta) \in \hat{\tau}_\delta$.

Therefore $\hat{\tau}_\delta$ contains the null linguistic pythagorean set and absolute linguistic pythagorean set.

Let $G_1(\delta), G_2(\delta) \in \hat{\tau}_\delta$. Then there exist $(G_1, \lambda), (G_2, \lambda) \in \hat{\tau}$ such that $\delta \in \lambda$. Since $\hat{\tau}$ is a linguistic pythagorean super hypersoft topology on $\underset{\wedge}{U}$, $(G_1, \lambda) \cap (G_2, \lambda) \in \hat{\tau}$. Let $(G_3, \lambda) = (G_1, \lambda) \cap (G_2, \lambda)$. Then $(G_3, \lambda) \in \hat{\tau}$. Note that $G_3(\delta) = G_1(\delta) \cap G_2(\delta)$. Since $(G_3, \lambda) \in \hat{\tau}$, it ollows that $G_3(\delta) \in \hat{\tau}_\delta$.

Therefore, $G_1(\delta) \cap G_2(\delta) \in \hat{\tau}_\delta$. Hence $\in \hat{\tau}_\delta$ is closed under finite intersections.

Let $\{G_i(\delta): i \in I\} \subseteq \hat{\tau}_\delta$. Then for every $i \in I$, there exists $(G_i, \lambda) \in \hat{\tau}$ such that $\delta \in \lambda$. Since $\hat{\tau}$ is a linguistic pythagorean super hypersoft topology on $\underset{\wedge}{U}$, $\bigcup_{i\in I}\{(G_i, \lambda): i \in I\} \in \hat{\tau}$. Let $(G, \lambda) = \bigcup_{i\in I}\{(G_i, \lambda): i \in I\}$. Then $(G, \lambda) \in \hat{\tau}$.

Note that $G(\delta) = \bigcup_{i\in I} G_i(\delta)$. Since $(G, \lambda) \in \hat{\tau}$, $G(\delta) \in \hat{\tau}_\delta$. Therefore, $\bigcup_{i\in I} G_i(\delta) \in \hat{\tau}_\delta$.

Thus, $\hat{\tau}_\delta = \{G(\delta): (G, \lambda) \in \hat{\tau}\}$ is a linguistic pythagorean topology on $\underset{\wedge}{U}$.

Definition 3.3: Let $(\underset{\wedge}{U}, \hat{\tau}, \lambda)$ be a linguistic pythagorean super hypersoft topological space over $\underset{\wedge}{U}$ and (G, λ) be a linguistic pythagorean super hypersoft set. The linguistic pythagorean super hypersoft interior of (G, λ) denoted by $\text{int}_{LPSH}(G, \lambda)$ is defined as the union of all linguistic pythagorean super hypersoft open subsets of (G, λ). That is, $\text{int}_{LPSH}(G, \lambda)$ is the largest linguistic pythagorean super hypersoft open set contained in (G, λ).

Theorem 3.1: Let $(\underset{\wedge}{U}, \hat{\tau}, \lambda)$ be a linguistic pythagorean super hypersoft topological space over $\underset{\wedge}{U}$ and $(G_1, \lambda), (G_2, \lambda) \in LPSH(\underset{\wedge}{U}, \lambda)$. Then the following properties hold

1. $\text{int}_{LPSH}(LPSH_\phi(\underset{\wedge}{U}, \lambda)) = LPSH_\phi(\underset{\wedge}{U}, \lambda)$ and $\text{int}_{LPSH}(LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)) = LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)$,
2. $\text{int}_{LPSH}(G_1, \lambda) \subseteq (G_1, \lambda)$,
3. (G_1, λ) is a linguistic pythagorean super hypersoft open set if and only if $\text{int}_{LPSH}(G_1, \lambda) = (G_1, \lambda)$,
4. $\text{int}_{LPSH}(\text{int}_{LPSH}(G_1, \lambda) = \text{int}_{LPSH}(G_1, \lambda)$,
5. If $(G_1, \lambda) \subseteq (G_2, \lambda)$, then $\text{int}_{LPSH}(G_1, \lambda) \subseteq \text{int}_{LPSH}(G_2, \lambda)$,
6. $\text{int}_{LPSH}((G_1, \lambda) \cap (G_2, \lambda)) = \text{int}_{LPSH}(G_1, \lambda) \cap \text{int}_{LPSH}(G_2, \lambda)$.

Proof

1. $\mathrm{int_{LPSH}LPSH_{\phi}}(\underset{\wedge}{\mathrm{U}},\lambda) = \cup\{(\mathrm{G}_1,\lambda) \in \hat{\tau}\colon (\mathrm{G}_1,\lambda) \subseteq \mathrm{LPSH_{\phi}}(\underset{\wedge}{\mathrm{U}},\lambda)\} = \cup\{\mathrm{LPSH_{\phi}}(\underset{\wedge}{\mathrm{U}},\lambda)\} = \mathrm{LPSH_{\phi}}(\underset{\wedge}{\mathrm{U}},\lambda)$.

$\mathrm{int_{LPSH}}(\mathrm{LPSH}_{\underset{\wedge}{\mathrm{U}}}(\underset{\wedge}{\mathrm{U}},\lambda)) = \cup\{(\mathrm{G}_1,\lambda) \in \hat{\tau}\colon (\mathrm{G}_1,\lambda) \subseteq (\mathrm{LPSH}_{\underset{\wedge}{\mathrm{U}}}(\underset{\wedge}{\mathrm{U}},\lambda))\} = \cup\{\hat{\tau}\} = (\mathrm{LPSH}_{\underset{\wedge}{\mathrm{U}}}(\underset{\wedge}{\mathrm{U}},\lambda))$.

2. $\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda)$ is the largest linguistic pythagorean super hypersoft open set contained in (G_1,λ).

Therefore $\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda) \subseteq (\mathrm{G}_1,\lambda)$.

3. Let (G_1,λ) be a linguistic pythagorean super hypersoft open set.

Since $\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda)$ is the largest linguistic pythagorean super hypersoft open set contained in (G_1,λ), it follows that

$$\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda_1) = (\mathrm{G}_1,\lambda_1).$$

Conversely, suppose that $\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda_1) = (\mathrm{G}_1,\lambda_1)$. Since $\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda_1)$ is a linguistic pythagorean super hypersoft open set, it follows that (G_1,λ_1) is also a linguistic pythagorean super hypersoft open set.

4. Let $\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda) = (\mathrm{G}_1,\lambda)$. Since (G_1,λ) is a linguistic pythagorean super hypersoft open set, $\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda) = (\mathrm{G}_1,\lambda)$.

Thus, $\mathrm{int_{LPSH}}\,(\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda)) = \mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda)$.

5. Let $(\mathrm{G}_1,\lambda) \subseteq (\mathrm{G}_2,\lambda)$. Since $\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda)$ is an open set contained in (G_1,λ), it follows that

$$\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda) \subseteq (\mathrm{G}_2,\lambda).$$

Also, since $\mathrm{int_{LPSH}}(\mathrm{G}_2,\lambda)$ is the largest linguistic pythagorean super hypersoft open set contained in (G_2,λ).

$$\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda) \subseteq \mathrm{int_{LPSH}}(\mathrm{G}_2,\lambda).$$

6. $\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda) \subseteq (\mathrm{G}_1,\lambda)$ and $\mathrm{int_{LPSH}}(\mathrm{G}_2,\lambda) \subseteq (\mathrm{G}_2,\lambda)$.

Hence $\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda) \cap \mathrm{int_{LPSH}}(\mathrm{G}_2,\lambda) \subseteq (\mathrm{G}_1,\lambda) \cap (\mathrm{G}_2,\lambda)$.

Since the largest linguistic pythagorean super hypersoft open set contained in $(\mathrm{G}_1,\lambda) \cap (\mathrm{G}_2,\lambda)$ is $\mathrm{int_{LPSH}}((\mathrm{G}_1,\lambda) \cap (\mathrm{G}_2,\lambda))$.

$$\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda) \cap \mathrm{int_{LPSH}}(\mathrm{G}_2,\lambda) \subseteq \mathrm{int_{LPSH}}((\mathrm{G}_1,\lambda) \cap (\mathrm{G}_2,\lambda)).$$

Conversely,

$\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda) \cap \mathrm{int_{LPSH}}(\mathrm{G}_2,\lambda) \subseteq \mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda)$ and $\mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda) \cap \mathrm{int_{LPSH}}(\mathrm{G}_2,\lambda) \subseteq \mathrm{int_{LPSH}}(\mathrm{G}_2,\lambda)$.

Therefore, $\mathrm{int_{LPSH}}((\mathrm{G}_1,\lambda) \cap (\mathrm{G}_2,\lambda)) \subseteq \mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda) \cap \mathrm{int_{LPSH}}(\mathrm{G}_2,\lambda)$.

Hence, $\mathrm{int_{LPSH}}((\mathrm{G}_1,\lambda) \cap (\mathrm{G}_2,\lambda)) = \mathrm{int_{LPSH}}(\mathrm{G}_1,\lambda) \cap \mathrm{int_{LPSH}}(\mathrm{G}_2,\lambda)$.

Example 3.3: Consider the attribute values from Example 3.1. Let $\hat{\tau} = \{(\mathrm{LPSH_{\phi}}(\underset{\wedge}{\mathrm{U}},\lambda)), (\mathrm{LPSH}_{\underset{\wedge}{\mathrm{U}}}(\underset{\wedge}{\mathrm{U}},\lambda)), (\mathrm{G}_1, \lambda), (\mathrm{G}_2, \lambda), (\mathrm{G}_3, \lambda), (\mathrm{G}_4, \lambda), (\mathrm{G}_5, \lambda)\}$ is a linguistic pythagorean super hypersoft topology on $\underset{\wedge}{\mathrm{U}}$. Define $\{s_1\ s_2\}, \emptyset$

$$(\mathrm{G}_5, \lambda) = \{<(\{s_1\ s_2\}, \emptyset), \{\frac{u_1}{(\text{dissatisfied,medium})}, \frac{u_2}{(\text{dissatisfied,satisfied})}, \frac{u_3}{(\text{dissatisfied,medium})}>\}.$$

Therefore

$$\mathrm{int_{LPSH}}(\mathrm{G}_5, \lambda) = (\mathrm{LPSH_{\phi}}(\underset{\wedge}{\mathrm{U}},\lambda)) \cup (\mathrm{G}_1, \lambda) \cup (\mathrm{G}_2, \lambda) \cup (\mathrm{G}_3, \lambda) \cup (\mathrm{G}_4, \lambda) = (\mathrm{G}_5, \lambda).$$

Definition 3.4: Let $(\underset{\wedge}{\mathrm{U}}, \hat{\tau}, \lambda)$ be a linguistic pythagorean super hypersoft topological space over $\underset{\wedge}{\mathrm{U}}$ and (G, λ) be a linguistic pythagorean super hypersoft set. The linguistic pythagorean super hypersoft closure of (G, λ) denoted by $\mathrm{cl_{LPSH}}(\mathrm{G}, \lambda)$ is defined as the linguistic pythagorean super hypersoft intersection of all linguistic pythagorean super hypersoft closed supersets of (G, λ). That is, $cl_{LPSH}(G, \lambda)$ is a smallest linguistic pythagorean super hypersoft closed set is containing (G, λ).

Proposition 3.3: Let $(\underset{\wedge}{U}, \hat{\tau}, \lambda)$ be a linguistic pythagorean super hypersoft topological space over $\underset{\wedge}{U}$. Then the following properties hold:

1. $(LPSH_{\phi}(\underset{\wedge}{U}, \lambda))$ and $(LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda))$ are linguistic pythagorean super hypersoft closed set over $\underset{\wedge}{U}$.
2. The intersection of any collection of linguistic pythagorean super hypersoft closed set is a linguistic pythagorean super hypersoft closed set over $\underset{\wedge}{U}$.
3. The union of any two linguistic pythagorean super hypersoft closed set is a linguistic pythagorean super hypersoft closed set over $\underset{\wedge}{U}$.

Proof

1. Since $(LPSH_{\phi}(\underset{\wedge}{U}, \lambda)) \in \hat{\tau}$, $(LPSH_{\phi}(\underset{\wedge}{U}, \lambda))^c = (LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda))$ is a linguistic pythagorean super hypersoft closed set, it follows that $(LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda))$ is a LPSH-closed set. Conversely, if $(LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)) \in \hat{\tau}$, then $(LPSH_{\phi}(\underset{\wedge}{U}, \lambda))$ is a linguistic pythagorean super hypersoft closed set.
2. Let $\{(G_i, \lambda) : i \in I\}$ be a collection of linguistic pythagorean super hypersoft closed sets, then $\bigcup_{i\in I}(G_i, \lambda)^c \in \hat{\tau}$. Thus $(\bigcap_{i\in I}(G_i, \lambda))^c \in \hat{\tau}$. Hence, $\bigcap_{i\in I}(G_i, \lambda)$ is a linguistic pythagorean super hypersoft closed set over $\underset{\wedge}{U}$.
3. Let (G_1, λ), (G_2, λ) be two linguistic pythagorean super hypersoft closed sets. Then $(G_1, \lambda)^c$, $(G_2, \lambda)^c \in \hat{\tau}$. Also, $((G_1, \lambda) \cup (G_2, \lambda))^c = (G_1, \lambda)^c \cap (G_2, \lambda)^c$. Therefore, $((G_1, \lambda) \cup (G_2, \lambda))^c \in \hat{\tau}$ and hence $(G_1, \lambda) \cup (G_2, \lambda)$ is a linguistic pythagorean super hypersoft closed set.

Theorem 3.2 Let $(\underset{\wedge}{U}, \hat{\tau}, \lambda)$ be a linguistic pythagorean super hypersoft space over $\underset{\wedge}{U}$ and let (G_1, λ), $(G_2, \lambda) \in LPSH((\underset{\wedge}{U}, \lambda)$. Then the following properties hold

1. $cl_{LPSH}(LPSH_{\phi}(\underset{\wedge}{U}, \lambda)) = LPSH_{\phi}(\underset{\wedge}{U}, \lambda))$ and $cl_{LPSH}(LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)) = (LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda))$
2. $(G_1, \lambda) \subseteq cl_{LPSH}(G_1, \lambda)$
3. (G_1, λ) is a linguistic pythagorean super hypersoft closed set if and only if $(G_1, \lambda) = cl_{LPSH}(G_1, \lambda)$
4. $cl_{LPSH}(cl_{LPSH}(G_1, \lambda)) = cl_{LPSH}(G_1, \lambda)$
5. If $(G_1, \lambda) \subseteq (G_2, \lambda)$, then $cl_{LPSH}(G_1, \lambda) \subseteq cl_{LPSH}(G_2, \lambda)$
6. $cl_{LPSH}((G_1, \lambda) \cup (G_2, \lambda)) = cl_{LPSH}(G_1, \lambda) \cup cl_{LPSH}(G_2, \lambda)$

Proof

1. $cl_{LPSH}(LPSH_{\phi}(\underset{\wedge}{U}, \lambda)) = \cap\{(G_1, \lambda) : LPSH_{\phi}(\underset{\wedge}{U}, \lambda) \subseteq (G_1, \lambda), (G_1, \lambda) \text{ is closed}\} = \cap\{LPSH_{\phi}(\underset{\wedge}{U}, \lambda)\} = LPSH_{\phi}(\underset{\wedge}{U}, \lambda)$.

$cl_{LPSH}(LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)) = \cap\{(G_1, \lambda) : LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda) \subseteq (G_1, \lambda), (G_1, \lambda) \text{ is closed}\} = \cap\{LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)\} = LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)$.

2. $cl_{LPSH}(G_1, \lambda)$ is the intersection of all linguistic pythagorean super hypersoft closed sets containing (G_1, λ). Since every closed set in this collection contains (G_1, λ), their intersection also contains (G_1, λ).

Therefore $(G_1, \lambda) \subseteq cl_{LPSH}(G_1, \lambda)$.

3. Let (G_1, λ) be a linguistic pythagorean super hypersoft closed set.

Since $cl_{LPSH}(G_1, \lambda)$ is a smallest linguistic pythagorean super hypersoft closed set contained in (G_1, λ), it follows that

$$cl_{LPSH}(G_1, \lambda) = (G_1, \lambda).$$

Conversely, suppose that $cl_{LPSH}(G_1, \lambda) = (G_1, \lambda)$. Since $cl_{LPSH}(G_1, \lambda)$ is a linguistic pythagorean super hypersoft closed set, (G_1, λ) is also a linguistic pythagorean super hypersoft closed set.

4. Let $cl_{LPSH}(G_1, \lambda) = (G_1, \lambda)$. Since (G_1, λ) is a linguistic pythagorean super hypersoft closed set, $cl_{LPSH}(G_1, \lambda) = (G_1, \lambda)$.

Thus, $cl_{LPSH}(cl_{LPSH}(G_1, \lambda)) = cl_{LPSH}(G_1, \lambda)$.

5. Let $(G_1, \lambda) \subseteq (G_2, \lambda)$. Since $(G_1, \lambda) \subseteq cl_{LPSH}(G_1, \lambda)$, it follows that

$$(G_2, \lambda) \subseteq cl_{LPSH}(G_1, \lambda).$$

Also, since $cl_{LPSH}(G_2, \lambda)$ is a smallest linguistic pythagorean super hypersoft closed set contained in (G_2, λ).

$$cl_{LPSH}(G_1, \lambda) \subseteq cl_{LPSH}(G_2, \lambda).$$

6. $(G_1, \lambda) \subseteq cl_{LPSH}(G_1, \lambda)$ and $(G_2, \lambda) \subseteq cl_{LPSH}(G_2, \lambda)$.

Therefore, $cl_{LPSH}(G_1, \lambda) \cup cl_{LPSH}(G_2, \lambda) \subseteq (G_1, \lambda) \cup (G_2, \lambda)$.

Since a smallest linguistic pythagorean super hypersoft closed set containing $(G_1, \lambda) \cup (G_2, \lambda)$ is $cl_{LPSH}((G_1, \lambda) \cup (G_2, \lambda))$.

$$cl_{LPSH}(G_1, \lambda) \cup cl_{LPSH}(G_2, \lambda) \subseteq cl_{LPSH}(G_1, \lambda) \cup (G_2, \lambda)).$$

Conversely,

$cl_{LPSH}(G_1, \lambda) \subseteq cl_{LPSH}(G_1, \lambda) \cup cl_{LPSH}(G_2, \lambda)$ and $cl_{LPSH}(G_2, \lambda) \subseteq cl_{LPSH}(G_1, \lambda) \cup cl_{LPSH}(G_2, \lambda)$.

Therefore, $cl_{LPSH}(G_1, \lambda) \cup cl_{LPSH}(G_2, \lambda) \subseteq cl_{LPSH}((G_1, \lambda) \cup (G_2, \lambda))$.

Hence, $cl_{LPSH}((G_1, \lambda) \cup (G_2, \lambda)) = cl_{LPSH}(G_1, \lambda) \cup cl_{LPSH}(G_2, \lambda)$.

Theorem 3.3: Let $(\underset{\wedge}{U}, \hat{\tau}, \lambda)$ be a linguistic pythagorean super hypersoft topological space over $\underset{\wedge}{U}$ and $(G, \lambda) \in LPSH(\underset{\wedge}{U}, \lambda)$, Then

1. $(cl_{LPSH}(G, \lambda))^c = int_{LPSH}((G, \lambda)^c)$,
2. $(int_{LPSH}(G, \lambda))^c = cl_{LPSH}((G, \lambda)^c)$.

Proof

1. $cl_{LPSH}(G, \lambda) = \cap\{(G, \lambda) : (G, \lambda)$ is closed and $(G, \lambda) \subseteq (G, \lambda)\}$. Taking complements on both sides, $(cl_{LPSH}(G, \lambda))^c = \cup\{(G, \lambda)^c : (G, \lambda)$ is closed and $(G, \lambda) \subseteq (G, \lambda)\}$. Since the complement of every closed set is open, $(G, \lambda)^c$ is a linguistic pythagorean super hypersoft open set.

Therefore, $(cl_{LPSH}(G, \lambda))^c$ is the union of all linguistic pythagorean super hypersoft open subsets of $(G, \lambda)^c$. By the definition of interior, $(cl_{LPSH}(G, \lambda))^c = int_{LPSH}((G, \lambda)^c)$.

2. Similarly, replacing (G, λ) by its complement $(G, \lambda)^c$, $(cl_{LPSH}((G, \lambda)^c))^c = (int_{LPSH}(G, \lambda))$. Taking complements on both sides $(int_{LPSH}(G, \lambda))^c = cl_{LPSH}((G, \lambda)^c)$.

Example 3.4: Consider the parameter values from Example 3.1

Let $\hat{\tau} = \{(LPSH_{\phi}(\underset{\wedge}{U}, \lambda)), (LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)), (G_1, \lambda), (G_2, \lambda), (G_3, \lambda), (G_4, \lambda), (G_5, \lambda)\}$ is a linguistic pythagorean super hypersoft topology on $\underset{\wedge}{U}$. Define

$$(G_5, \lambda) = \{<(\{s_1\ s_2\}, \emptyset), \{\frac{u_1}{(\text{dissatisfied,medium})}, \frac{u_2}{(\text{dissatisfied,satisfied})}, \frac{u_3}{(\text{dissatisfied,medium})}>\}.$$

Therefore

$$int_{LPSH}(G_5, \lambda) = (LPSH_{\phi}(\underset{\wedge}{U}, \lambda)) \cup (G_1, \lambda) \cup (G_2, \lambda) \cup (G_3, \lambda) \cup (G_4, \lambda) = (G_5, \lambda).$$

Definition 3.5: Let $(\underset{\wedge}{U}, \hat{\tau}, \lambda)$ be a linguistic pythagorean super hypersoft topological space over $\underset{\wedge}{U}$ and $Q \subseteq \hat{\tau}$. Q is called a linguistic pythagorean super hypersoft basis for the linguistic pythagorean super hypersoft topology $\hat{\tau}$. If every element of $\hat{\tau}$ can be written as linguistic pythagorean super hypersoft union of elements of Q.

Proposition 3.4: Let $(\underset{^}{U}, \hat{\tau}, \lambda)$ be linguistic pythagorean super hypersoft topological space over $\underset{^}{U}$ and Q be a linguistic pythagorean super hypersoft basis for $\hat{\tau}$. Then $\hat{\tau}$ equals the collection of linguistic pythagorean super hypersoft union of elements of Q.

Proof

By the definition of linguistic pythagorean super hypersoft basis

$$\forall\ (G, \lambda) \in \hat{\tau},\ \bigcup_{i\in I}\{(G_i, \lambda)\} \subseteq Q$$

such that

$$(G, \lambda) = \bigcup_{i\in I}(G_i, \lambda).$$

That is, $\hat{\tau} \subseteq \{\bigcup_{i\in I}(G_i, \lambda) : (G_i, \lambda) \in Q\}$.

Conversely, each element of Q belongs to $\hat{\tau}$, and since $\hat{\tau}$ is closed under arbitrary unions, every union of elements of Q also belongs to $\hat{\tau}$

$$\{\bigcup_{i\in I}(G_i, \lambda) : (G_i, \lambda) \in Q\} \subseteq \hat{\tau}.$$

Therefore, $\hat{\tau} = \{\bigcup_{i\in I}(G_i, \lambda) : (G_i, \lambda) \in Q\}$.

Example 3.5: Consider Example 3.3 Then,

$\hat{\tau} = \{(LPSH_{\phi}(\underset{^}{U}, \lambda)), (LPSH_{\underset{^}{U}}(\underset{^}{U}, \lambda)), (G_1, \lambda), (G_2, \lambda), (G_3, \lambda), (G_4, \lambda), (G_5, \lambda)\}$ is a linguistic pythagorean super hypersoft topology on $\underset{^}{U}$. Define

$$(G_4, \lambda) = \{<(\{s_2\}, \{t_1\}), \{\frac{u_1}{(\text{medium,medium})}, \frac{u_2}{(\text{V.dissatisfied,satisfied})}, \frac{u_3}{(\text{dissatisfied,satisfied})}>\}.$$

Therefore,

$$Q = \{(LPSH_{\phi}(\underset{^}{U}, \lambda)), (LPSH_{\underset{^}{U}}(\underset{^}{U}, \lambda)), (G_1, \lambda), (G_2, \lambda), (G_3, \lambda), (G_5, \lambda)\}$$

is a linguistic pythagorean super hypersoft basis for the linguistic pythagorean super hypersoft topology $\hat{\tau}$.

Theorem 3.4: Let $(\underset{^}{U}, \hat{\tau}, \lambda)$ be a linguistic pythagorean super hypersoft topological space over $\underset{^}{U}$ and (G, λ) be a linguistic pythagorean super hypersoft set over $\underset{^}{U}$. Define

$$\hat{\tau}_{(G,\lambda)} = \{(G, \lambda) \cap (G_i, \lambda) : (G_i, \lambda) \in \hat{\tau} \text{ for } i \in I\}$$

is a linguistic pythagorean super hypersoft topology on the linguistic pythagorean super hypersoft subset (G, λ) relative to the parameter set, super hypersoft parameter family λ.

Proof: $(LPSH_{\phi}(\underset{^}{U}, \lambda)), (LPSH_{\underset{^}{U}}(\underset{^}{U}, \lambda)) \in \hat{\tau}_{(G,\lambda)}$. Moreover,

$$\bigcap_{i\in I}(G, \lambda)\ \cap\ (G_i, \lambda) = (G, \lambda) \cap (\bigcap_{i\in I}(G_i, \lambda)),$$

and

$$\bigcup_{i\in I}(G, \lambda)\ \cap\ (G_i, \lambda) = (G, \lambda) \cap (\bigcup_{i\in I}(G_i, \lambda)).$$

Thus, the linguistic pythagorean super hypersoft union of linguistic pythagorean super hypersoft sets in $\hat{\tau}_{(G,\lambda)}$ belongs to $\hat{\tau}_{(G,\lambda)}$ and the finite linguistic pythagorean super hypersoft intersection of linguistic pythagorean super hypersoft sets in $\hat{\tau}_{(G,\lambda)}$ belongs to $\hat{\tau}_{(G,\lambda)}$.

Hence, $\hat{\tau}_{(G,\lambda)}$ is a linguistic pythagorean super hypersoft topology on (G, λ).

Definition 3.6: Let $(\underset{^}{U}, \hat{\tau}, \lambda)$ be a linguistic pythagorean super hypersoft topological space over $\underset{^}{U}$ and (G, λ) be a linguistic pythagorean super hypersoft set over $\underset{^}{U}$. Then the linguistic pythagorean super hypersoft topology

$$\hat{\tau}_{(G,\lambda)} = \{(G, \lambda) \cap (G_i, \lambda): (G_i, \lambda) \in \hat{\tau} \text{ for } i \in I\}$$

is called the linguistic pythagorean super hypersoft subspace topology and $((G, \lambda), \hat{\tau}_{(G,\lambda)}, \lambda)$ is called a linguistic pythagorean super hypersoft subspace of $(\underset{^}{U}, \hat{\tau}, \lambda)$.

4. LPSHT space Algorithm to solve MCDM Problem

Step 1: Consider a universe of $\underset{\wedge}{U}$ and a super-attribute set be $\lambda = \{$Ᵽ$(A_1) \times . . . . . \times$ Ᵽ$(A_n)\}$.

Step 2: Construct the linguistic pythagorean super hypersoft set (G, λ) over $\underset{\wedge}{U}$, corresponding to the Super attributes in λ.

Step 3: Define linguistic pythagorean super hypersoft topology $\hat{\tau}$, (G_1, λ), (G_2, λ), (G_3, λ) and (G_4, λ) are open linguistic pythagorean super hypersoft set in $\hat{\tau}$.

Step 4: Compute the aggregated linguistic pythagorean super hypersoft sets (G_1, λ), (G_2, λ), (G_3, λ) and (G_4, λ) in $\hat{\tau}$.

Step 5: Define a linguistic pythagorean score as

$$\text{Score} = w(\text{membership} - \text{Non-membership})$$

where w represents linguistic terms,

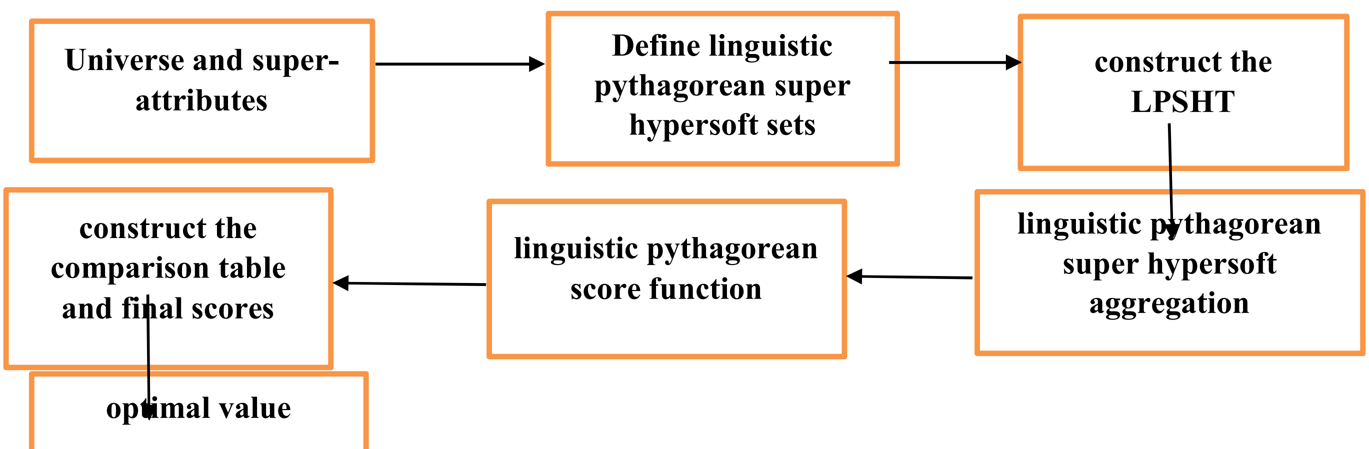
w(Extremely dissatisfied) = 0.1, w(dissatisfied) = 0.2, w(medium) = 0.5, w(satisfied) = 0.7 w(Extremely satisfied) = 0.9.

Step 6: Construct the comparison table for the resultant linguistic pythagorean super hypersoft set and compute the row sum D_j and column sum C_j for each alternative $u_j \in \underset{\wedge}{U}$. Calculate the score value for each alternative u_j using

$$O_j = D_j - C_j, \text{ for each element } u_j \in \underset{\wedge}{U}.$$

Step 7: Select the optimal alternative u_j such that $u_j = \arg\max \{O_j\}$. finally, list the alternatives with highest membership $(\mu_{G(\delta)}(u))$ value. The largest membership $(\mu_{G(\delta)}(u))$, will represent the positive ideal alternative.

Figure 4.1: Graphical representation of proposed LPSHT algorithm.



4.1 Illustrative Example

A doctor intends to diagnose three patients, $\underset{\wedge}{U} = \{u_1, u_2, u_3\}$ for the disease oral cancer. The diagnosis involves uncertainty due to overlapping groups of symptoms and causes represented by super-attributes. Let Ᵽ(A_1) denote the power set of the symptom attribute values {red patch, a lump, swelling of the jaw}, and let Ᵽ(A_2) denote the power set of the cause attribute values {tobacco use, alcohol use, HPV infection}. Since the assessment of patients involves uncertainty and hesitation in evaluating these super-attributes, classical decision-making models are inadequate to represent both the membership and non-membership information simultaneously.

Therefore, a linguistic pythagorean super hypersoft topological space is employed to model the uncertainty and identify the patients who are likely to be affected by Oral Cancer.

Step: 1

$\underset{\wedge}{U}$ = {u_1, u_2, u_3} be a universal set and a super-attribute set be λ ={$P(A_1) \times P(A_2)$}, where A_1: symptoms (a lump, swelling of the jaw) and A_2: causes (alcohol use, HPV infection). Let $P(A_1)$ = {ϕ, {a lump}, {swelling of the jaw}, {a lump, swelling of the jaw}}

$P(A_2)$ = {ϕ, {alcohol use}, {HPV infection}, {alcohol use, HPV infection}}

Step 2: Medical experts provide their evaluations in the form of LPSHSs ((G_1, λ), (G_2, λ), (G_3, λ) and (G_4, λ).

Step 3: Construct the linguistic pythagorean super hypersoft topology. The goal is to create topology such that these sets are open sets.

$\hat{\tau}$ = {($LPSH_{\phi}(\underset{\wedge}{U}, \lambda)$), ($LPSH_{\underset{\wedge}{U}}(\underset{\wedge}{U}, \lambda)$), ($G_1$, λ), ($G_2$, λ), ($G_3$, λ), ($G_4$, λ)}

where (G_1, λ), (G_2, λ), (G_3, λ), (G_4, λ) are defined as follows

Table 4.1: Form of (G_1, λ)

(G_1, λ)	**u_1**	**u_2**	**u_3**
({a lump}, {HPV infection}) = h_1	(satisfied, dissatisfied)	(satisfied, extremely dissatisfied)	(medium, dissatisfied)
({swelling of the jaw}, {HPV infection}) = h_2	(satisfied, medium)	(satisfied, medium)	(satisfied, dissatisfied)
({ϕ}, {alcohol use}) = h_3	(satisfied, dissatisfied)	(medium, dissatisfied)	(satisfied, dissatisfied)

Table 4.2: Form of (G_2, λ)

(G_2, λ)	**u_1**	**u_2**	**u_3**
({ϕ}{alcohol use, HPV infection}) = h_4	(satisfied, dissatisfied)	(medium, dissatisfied)	(medium, dissatisfied)
({a lump, swelling of the jaw}, {alcohol use}) = h_5	(medium, medium)	(satisfied, dissatisfied)	(satisfied, extremely dissatisfied)
({a lump}, {alcohol use, HPV infection}) = h_6	(medium, dissatisfied)	(medium, medium)	(medium, extremely dissatisfied)

Table 4.3: Form of (G_3, λ)

(G_3, λ)	**u_1**	**u_2**	**u_3**
({a lump}, {alcohol use}) = h_7	(medium, extremely dissatisfied)	(satisfied, medium)	(medium, medium)
({swelling of the jaw}, {alcohol use, HPV infection}) = h_8	(medium, medium)	(satisfied, dissatisfied)	(dissatisfied, extremely dissatisfied)
({swelling of the jaw},{alcohol use}) = h_9	(satisfied, medium)	(medium, dissatisfied)	(medium, dissatisfied)

Table 4.4: Form of (G_4, λ)

(G_4, λ)	u_1	u_2	u_3
({a lump, swelling of the jaw}, {alcohol use, HPV infection}) = h_{10}	(satisfied, dissatisfied)	(satisfied, medium)	(medium, dissatisfied)
({a lump, swelling of the jaw}, {ϕ}) = h_{11}	(satisfied, medium)	(satisfied, extremely dissatisfied)	(extremely satisfied, dissatisfied)
({a lump, swelling}, {HPV infection})= h_{12}	(medium, dissatisfied)	(dissatisfied, dissatisfied)	(dissatisfied, extremely dissatisfied)

Step: 4

Compute the aggregated linguistic pythagorean super hypersoft sets (G_1, λ), (G_2, λ), (G_3, λ) and (G_4, λ) in $\hat{\tau}$.

Table 4.5: Comparison table of $(G_1, \lambda) \vee (G_2, \lambda) \vee (G_3, \lambda) \vee (G_4, \lambda)$

$(G_1, \lambda) \vee (G_2, \lambda) \vee (G_3, \lambda) \vee (G_4, \lambda)$	u_1	u_2	u_3
$h_1 \times h_4 \times h_7 \times h_{10}$	(satisfied, extremely dissatisfied)	(satisfied, extremely dissatisfied)	(medium, dissatisfied)
$h_2 \times h_5 \times h_8 \times h_{11}$	(satisfied, medium)	(satisfied, extremely dissatisfied)	(extremely satisfied, extremely dissatisfied)
$h_3 \times h_6 \times h_9 \times h_{12}$	(satisfied, dissatisfied)	(medium, dissatisfied)	(satisfied, extremely dissatisfied)

Step: 5

Define a linguistic pythagorean Score as

Score = w(membership – Non-membership)

w(extremely dissatisfied) = 0.1, w(dissatisfied) = 0.2, w(medium) = 0.5, w(satisfied) = 0.7 w(extremely satisfied) = 0.9.

Step: 6

Table 4.6: Score value

$(G_1, \lambda) \vee (G_2, \lambda) \vee (G_3, \lambda) \vee (G_4, \lambda)$	u_1	u_2	u_3	row (D_j)
$h_1 \times h_4 \times h_7 \times h_{10}$	0.6	0.6	0.3	1.5
$h_2 \times h_5 \times h_8 \times h_{11}$	0.2	0.6	0.8	1.6
$h_3 \times h_6 \times h_9 \times h_{12}$	0.5	0.3	0.6	1.4
column (C_j)	1.3	1.5	1.7	

The score O_j for each u_j ($j = 1, 2, 3$) is computed as: $O_j = D_j - C_j$.

Patients **Score**

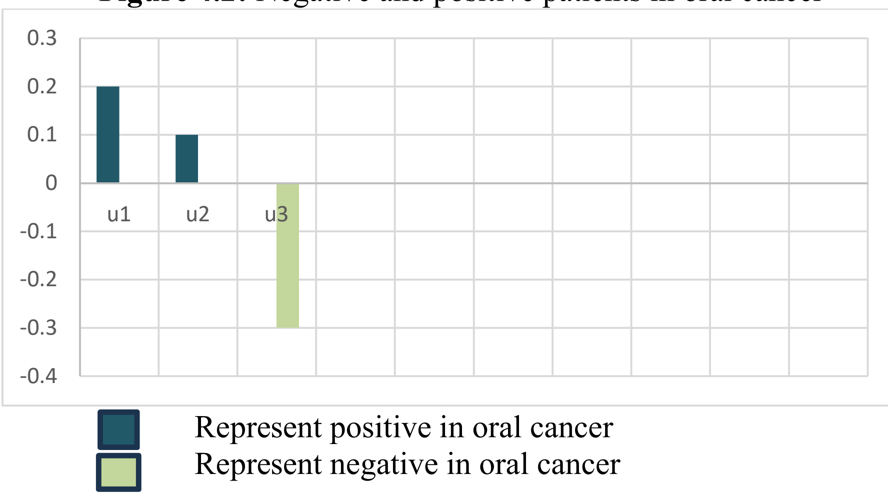
$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.1 \\ -0.3 \end{bmatrix}$$

Step:7

The optimal alternative u_j = arg max $\{O_j\}$ = row (D_j) – column (C_j), patient u_1 and u_2 is most likely to have oral cancer because it has the positive score value.

$$\begin{array}{ccccc} \textbf{Patients} & & \textbf{Score} & & \textbf{Result} \\ \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} & = & \begin{bmatrix} 0.2 \\ 0.1 \\ -0.3 \end{bmatrix} & = & \begin{bmatrix} \text{positive} \\ \text{positive} \\ \text{negative} \end{bmatrix} \end{array}$$

Figure 4.2: Negative and positive patients in oral cancer



Result: The outcomes of the linguistic pythagorean super hypersoft topology algorithm indicate a clear improvement in diagnostic. Precision when compared with traditional methods. The LPSET algorithm was constructed where the sets (G_1, λ), (G_2, λ), (G_3, λ), (G_4, λ) were considered as open linguistic pythagorean super hypersoft sets. By applying the defined disjunctive operation, the resultant linguistic pythagorean super hypersoft set in Table 4.5 was derived. A comparison table was then formulated to evaluate the alternatives $(G_1, \lambda) \vee (G_2, \lambda) \vee (G_3, \lambda) \vee (G_4, \lambda)$. The score values were computed in Table 4.6 represent the row and column sums, respectively. The results revealed that Patient u_1 and u_2 attained the highest score and that shown in figure 4.2, thereby establishing it as the most suitable method for solving oral cancer.

Conclusion: In this article linguistic pythagorean super hypersoft topology was introduced to model linguistic pythagorean membership and linguistic pythagorean non-membership information in generalized topological spaces using concepts such as interior, closure, basis, and subspaces. The proposed framework was applied to decision-making problems through the LDHSHT algorithm, demonstrating its effectiveness and potential applications in multi-criteria decision-making (MCDM), medical diagnosis, and AI-based predictive systems.

Reference

[1] Azzam, et. al, "operations on complex Neutrosophic Soft Sets and their topological spaces: A new approach with applications to decision-making, European Journal of pure and applied Mathematics" ISSN 1307-5543, article no 7171, issue 4, vol.18, 2025.

[2] Harish Garg, “Linguistic Pythagorean fuzzy sets and its applications in multiattribute decision-making process, research article” DOI: 10.1002/int.21979.

[3] Harish Garg, et. al, “Spherical Fuzzy Soft Topology and Its Application in Group Decision-Making Problems, Hindawi” https://doi.org/10.1155/2022/1007133, 2022.

[4] Kaviyarasu M, et.al, “Neutrosophic Hypersoft Topological Framework for Agricultural Decision-Making, european journal of pure and applied mathematics” Vol. 18, Issue 2, Article Number 5905, ISSN 1307-5543 , 2025.

[5] Sathiyapriya Bangarusamy, et.al, “Linguistic Pythagorean Hypersoft Set with Application: Decision Making in Medical Diagnosis and Treatment, Neutrosophic sets and systems” Vol. 87, 2025.

[6] Suman Das, et. al, Neutrosophic Simply Soft Open Set in Neutrosophic Soft Topological Space, Neutrosophic sets and systems, Vol. 38, 2020.

[7] Takaaki Fujita, et.al, “Short Note of Linguistic IndetermSoft Set and Linguistic IndetermHyperSoft Set, Research Gate” DOI: 10.13140/RG.2.2.13986.54721, 2025.

[8] Takaaki Fujita, Florentin Smarandache, “Harnessing Quantum Superposition in Soft Set Theory: Introducing Quantum Hypersoft and SuperHyperSoft Sets, european journal of pure and applied mathematics”, ISSN: 1307-5543 Issue 3, Vol. 18, 2025.

[9] Zadeh, “Fuzzy Sets, Information and Computation”, vol. 8, pp. 338-353, 1965.

[10] Zanyar A. Ameen, et. al, “Congruence Representations via Soft Ideals in Soft Topological Spaces, MDPI” https://doi.org/10.3390/axioms12111015, 2023.