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NEUTROSOPHIC $\alpha\pi$ g HOMEOMORPHISM IN NEUTROSOPHIC TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce the concepts of Neutrosophic $\alpha\pi$ g Homeomorphisms and Neutrosophic $i-\alpha\pi$ g homeomorphisms. Further, we study some of their properties.

Key words

Neutrosophic Topological spaces, Neutrosophic α Closed set, Neutrosophic $\alpha\pi$ g Closed sets, Neutrosophic $\alpha\pi$ g homeomorphisms and Neutrosophic $i-\alpha\pi$ g homeomorphisms.

I. Introduction

The concept of fuzzy sets, introduced by zadeh[15] in 1965, allow each element to have a degree of membership. This concept was expanded by K. Atanassov [1] in 1986 with the introduction of Intuitionistic Fuzzy sets, which assign both a degree of membership and a degree of non-membership to each element. Florentin Samarandache[4] introduced Neutrosophic Sets as a further generalization, which adds more flexibility. Later, A.A. Salama and S.A. Albowi[5] extended the idea by developing Neutrosophic Topological Spaces.

In this paper we introduce of Neutrosophic $\alpha\pi$ g homeomorphisms and Neutrosophic $i-\alpha\pi$ g homeomorphisms.

II. Preliminaries

Definition 2.1:

Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x) and \nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A.

A neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ can be identified as an ordered triple $\langle \mu_A, \sigma_A, \nu_A \rangle$ in $]^-0,1^+[$ on X.

Definition 2.2:

For any two neutrosophic sets $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle : x \in X\}$ we may have,

1. $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x), \nu_A(x) \ge \nu_B(x) \forall x \in X$
2. $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$
3. $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$

4. $A^C = \{\langle x, \nu_A(x), 1-\sigma_A(x), \mu_A(x)\rangle \mid x \in X\}$

Definition 2.3:

A Neutrosophic topology (NT) on a non-empty set X is a family τ_N of neutrosophic subsets in X satisfies the following axioms:

(NT1) $0_N, 1_N \in \tau_N$

(NT2) $W_1 \cap W_2 \in \tau_N$ for any $W_1, W_2 \in \tau_N$

(NT3) $\cup W_j \in \tau_N \forall \{W_j : j \in I\} \subseteq \tau_N$

In this case the pair (X, τ_N) is a neutrosophic topological space (NTS) and any neutrosophic set in τ is known as a neutrosophic open set (NOS) in X. A neutrosophic set A is a neutrosophic Closed set (NCS) if and only if complement A^C is a neutrosophic open set in X.

Here the empty set 0_N and the whole set 1_N may be defined as follows:

(01) $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$

(11) $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$

Definition 2.4:

Let (X, τ_N) be a NTS and $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ be a NS in X. Then the neutrosophic interior and the neutrosophic closure of A are defined by

$NInt(A) = \cup\{H : H$ is an NOS in X $H \subseteq A\}$

$NCl(A) = \cap\{M : M$ is an NCS in X $A \subseteq M\}$

Definition 2.5:

An NS $A = \langle x, \mu_A, \sigma_A, \nu_A\rangle$ in an NTS (X, τ_N) is said to be an

(i) Neutrosophic regular closed set(NRCS in short) if $A = Ncl(N\text{int}(A))$,

(ii) Neutrosophic α closed set(NαCS in short) if $Ncl(N\text{int}(Ncl(A))) \subseteq A$,

(iii) Neutrosophic semi closed(NSCS in short) if $N\text{int}(Ncl(A)) \subseteq A$,

(iv)Neutrosophic Pre closed(NPCS in Short) if $Ncl(N\text{int}(A)) \subseteq A$,

(v) Neutrosophic Generalized closed set(NGCS in short) if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is an NOS in (X, τ_N),

(vi) Neutrosophic Generalized Semi closed set(NGSCS in short) if $Nscl(A) \subseteq U$ whenever $A \subseteq U$ and U is an NOS in (X, τ_N),

(vii) Neutrosophic Generalized pre closed set(NGPCS in short) if $Npcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an NOS in (X, τ_N),

The complements of the above mentioned Neutrosophic closed sets are called their Neutrosophic Open sets.

Definition 2.6:

An NS $A = \langle x, \mu_A, \sigma_A, \nu_A\rangle$ in an NTS (X, τ_N) is said to be an Neutrosophic π Open set (NπOS in short) if A is a finite union of Neutrosophic Regular open sets.

Definition 2.7:

Let A be an NTS (X, τ_N). Then the α-interior of A ($\alpha\text{int}(A)$ in short) and the α-closure of A ($\alpha cl(A)$ in short) are defined as

$N\alpha\,\text{int}(A) = \cup\{\ \{H : H$ is an $N\alpha$ OS in X $H \subseteq A\}$,

$N\alpha cl(A) = \cap\{M : M$ is an $N\alpha$ CS in X $A \subseteq M\ \}$.

Definition 2.8:

Let $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$ in an NTS (X, τ_N) is said to be an

i. $N\alpha cl(A) = A \cup Ncl(N\,\text{int}(Ncl(A)))$,
ii. $N\alpha\,\text{int}(A) = A \cap N\,\text{int}(Ncl(N\,\text{int}(A)))$.

Definition 2.9:

An NS A in (X, τ_N) is said to be an Neutrosophic $\alpha\pi$ Generalized closed set (N$\alpha\pi$GCS in short) if $N\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\ \pi\ OS$ in (X, τ_N). Here the family of all N$\alpha\pi$GCS of an NTS (X, τ_N) is denoted by N$\alpha\pi$GCS (X, τ_N).

Definition 2.10:

A mapping $g_N : (X, \tau_N) \to (Y, \eta_N)$ is said to be

i. Neutrosophic Continuous mapping if $g_N^{-1}(V)$ is Neutrosophic closed set in (X, τ_N) for every Neutrosophic closed set V of (Y, η_N),
ii. Neutrosophic Pre Continuous mapping if $g_N^{-1}(V)$ is Neutrosophic Pre closed set in (X, τ_N) for every Neutrosophic closed set V of (Y, η_N),
iii. Neutrosophic Semi Continuous mapping if $g_N^{-1}(V)$ is Neutrosophic Semi closed set in (X, τ_N) for every Neutrosophic closed set V of (Y, η_N),
iv. Neutrosophic Regular Continuous mapping if $g_N^{-1}(V)$ is Neutrosophic Regular closed set in (X, τ_N) for every Neutrosophic closed set V of (Y, η_N),
v. Neutrosophic α Continuous mapping if $g_N^{-1}(V)$ is Neutrosophic α closed set in (X, τ_N) for every Neutrosophic closed set V of (Y, η_N).

Definition 2.11:

A mapping $g_N : (X, \tau_N) \to (Y, \eta_N)$ is said to be

i. Neutrosophic Closed mapping if $g_N(V)$ is Neutrosophic closed in (Y, η_N) for every Neutrosophic closed V of (X, τ_N).
ii. Neutrosophic Pre Closed mapping if $g_N(V)$ is Neutrosophic Pre closed in (Y, η_N) for every Neutrosophic closed V of (X, τ_N).
iii. Neutrosophic Semi Closed mapping if $g_N(V)$ is Neutrosophic Semi closed in (Y, η_N) for every Neutrosophic closed V of (X, τ_N).
iv. Neutrosophic Semi Pre Closed mapping if $g_N(V)$ is Neutrosophic Semi Pre closed in (Y, η_N) for every Neutrosophic closed V of (X, τ_N).
v. Neutrosophic α Closed mapping if $g_N(V)$ is Neutrosophic α closed in (Y, η_N) for every Neutrosophic closed V of (X, τ_N).

vi. Neutrosophic Regular Closed mapping if $g_N(V)$ is Neutrosophic Regular closed in (Y,η_N) for every Neutrosophic closed V of (X,τ_N).

vii. Neutrosophic Generalized Closed mapping if $g_N(V)$ is Neutrosophic Generalized closed in (Y,η_N) for every Neutrosophic closed V of (X,τ_N).

viii. Neutrosophic Generalized Semi Closed mapping if $g_N(V)$ is Neutrosophic Generalized Semi closed in (Y,η_N) for every Neutrosophic closed V of (X,τ_N).

Definition 2.12:

Let g_N be a bijective mapping from an NTS (X,τ_N) into an NTS (Y,η_N). Then g_N is said to be an

i. Neutrosophic homeomorphisms if g_N and g_N^{-1} are Neutrosophic continuous mappings,

ii. Neutrosophic α homeomorphisms if g_N and g_N^{-1} are Neutrosophic α continuous mappings,

iii. Neutrosophic Generalized homeomorphisms if g_N and g_N^{-1} are Neutrosophic Generalized continuous mappings,

iv. Neutrosophic Generalized Semi homeomorphisms if g_N and g_N^{-1} are Neutrosophic Generalized Semi continuous mappings,

v. Neutrosophic Generalized Semi Pre homeomorphisms if g_N and g_N^{-1} are Neutrosophic Generalized Semi Pre continuous mappings.

Definition 2.13:

An NTS (X,τ_N) is said to be an Neutrosophic $\alpha\pi$ a$T_{1/2}$ (N$\alpha\pi$ a$T_{1/2}$ in short) space if every $N\alpha\pi GCS$ in X is an NCS in X.

Neutrosophic $\alpha\pi g$ Homeomorphisms in Neutrosophic Topological Spaces

In this section we introduce Neutrosophic $\alpha\pi g$ homeomorphisms and study some of its properties.

Definition 3.1:

A bijective mapping $g_N:(X,\tau_N)\to(Y,\eta_N)$ is called an Neutrosophic $\alpha\pi g$ homeomorphism (N$\alpha\pi g$ homeomorphism in short) if g_N and g_N^{-1} are N$\alpha\pi$ g continuous mappings.

Example 3.2:

Let $X=\{a,b\}$, $Y=\{u,v\}$ and $W_1=\langle x,(0.5,0.6,0.3),(0.2,0.5,0.4)\rangle$ $W_2=\langle y,(0.4,0.5,0.6),(0.3,0.5,0.5)\rangle$. Then $\tau_N=\{0_N,W_1,1_N\}$ and $\eta_N=\{0_N,W_2,1_N\}$ are NTSs on X and Y respectively. Define a bijective mapping $g_N:(X,\tau_N)\to(Y,\eta_N)$ by $f(a)=u$ and $f(b)=v$. Then g_N is an N$\alpha\pi g$ continuous mapping and g_N^{-1} is also an N$\alpha\pi g$ continuous mapping. Therefore the bijective mapping g_N is an N$\alpha\pi g$ homeomorphism.

Theorem 3.3:

Let $g_N:(X,\tau_N)\to(Y,\eta_N)$ be a bijective mapping from an NTS X into an NTS Y. Then the following conditions are equivalent:

i. g_N is an Neutrosophic homeomorphisms,

ii. g_N is an Neutrosophic mapping and g_N is an Neutrosophic open mapping,

iii. g_N and g_N^{-1} are Neutrosophic continuous mappings.

Proof:

(i) ⇒ (ii): It is obviously true.

(ii) ⇒ (iii): Let g_N is an Neutrosophic open mapping. That is $g_N(V)$ is an NOS in Y for each NOS V in X. Now define a mapping g_N^{-1}: $(Y,\eta) \to (X,\tau)$. By hypotheses, for every NOS A in X, we have g_N^{-1}(V) is an NOS in Y. Hence g_N^{-1} is an Neutrosophic continuous mapping. That is g_N and g_N^{-1} are Neutrosophic continuous mappings.

(iii)⇒(i): Let g_N and g_N^{-1} are Neutrosophic continuous mapping. Since g_N^{-1}: $(Y,\eta) \to (X,\tau)$ is an Neutrosophic continuous mappings, $g_N : (X,\tau_N) \to (Y,\eta_N)$ is an Neutrosophic open mapping. Hence g_N is an Neutrosophic homeomorphism.

Theorem 3.4:

Every Neutrosophic Homeomorphism is an N $\alpha\pi g$ homeomorphism but not conversely.

Proof:

Let $g_N : (X,\tau_N) \to (Y,\eta_N)$ be an Neutrosophic homeomorphism. Then g_N and g_N^{-1} are Neutrosophic continuous mappings. This implies g_N and g_N^{-1} are N $\alpha\pi g$ continuous mappings. That is g_N is an N $\alpha\pi g$ homeomorphism.

Example 3.5:

Let $X = \{a,b\}$, $Y = \{u,v\}$ and $W_1 = \langle x,(0.4,0.5,0.6),(0.3,0.5,0.7)\rangle$ $W_2 = \langle y,(0.7,0.2,0.1),(0.8,0.9,0.2)\rangle$. Then $\tau_N = \{0_N, W_1, 1_N\}$ and $\eta_N = \{0_N, W_2, 1_N\}$ are NTSs on X and Y respectively. Define a bijective mapping $g_N : (X,\tau_N) \to (Y,\eta_N)$ by $f(a) = u$ and $f(b) = v$. Then g_N^{-1}(V) is not an NCS in X. Therefore g_N is not an Neutrosophic continuous mapping. Therefore the bijective mapping g_N is not an Neutrosophic Homeomorphism. But g_N is an N $\alpha\pi g$ continuous mapping and g_N^{-1} is also an N $\alpha\pi g$ continuous mapping. Therefore the bijective mapping g_N is an N $\alpha\pi g$ homeomorphism.

Theorem 3.6:

Every Neutrosophic α Homeomorphism is an N $\alpha\pi g$ homeomorphism but not conversely.

Proof:

Let $g_N : (X,\tau_N) \to (Y,\eta_N)$ be an Neutrosophic α homeomorphism. Then g_N and g_N^{-1} are Neutrosophic α continuous mappings. This implies g_N and g_N^{-1} are N $\alpha\pi g$ continuous mappings. That is g_N is an N $\alpha\pi g$ homeomorphism.

Example 3.7:

Let $X = \{a,b\}$, $Y = \{u,v\}$ and $W_1 = \langle x,(0.3,0.5,0.7),(0.2,0.5,0.7)\rangle$ $W_2 = \langle y,(0.4,0.3,0.2),(0.6,0.8,0.3)\rangle$. Then $\tau_N = \{0_N, W_1, 1_N\}$ and $\eta_N = \{0_N, W_2, 1_N\}$ are NTSs on X and Y respectively. Define a bijective mapping $g_N : (X,\tau_N) \to (Y,\eta_N)$ by $f(a) = u$ and $f(b) = v$. Then g_N^{-1}(V) is not an N α CS in X. Therefore g_N is not an Neutrosophic α continuous mapping. Therefore the bijective mapping g_N is not an Neutrosophic α Homeomorphism. But g_N is an N $\alpha\pi g$ continuous mapping and g_N^{-1} is also an N $\alpha\pi g$ continuous mapping. Therefore the bijective mapping g_N is an N $\alpha\pi g$ homeomorphism.

Theorem 3.8:

Every Neutrosophic Generalized Homeomorphism is an N $\alpha\pi g$ homeomorphism but not conversely.

Proof:

Let $g_N : (X,\tau_N) \to (Y,\eta_N)$ be an Neutrosophic Generalized homeomorphism. Then g_N and g_N^{-1} are

Neutrosophic Generalized continuous mappings. This implies g_N and g_N^{-1} are N $\alpha\pi g$ continuous mappings. That is g_N is an N $\alpha\pi g$ homeomorphism.

Example 3.9:

Let $X=\{a,b\}$, $Y=\{u,v\}$ and $W_1=\langle x,(0.3,0.5,0.2),(0.2,0.5,0.1)\rangle$ $W_2=\langle y,(0.4,0.5,0.2),(0.2,0.5,0.2)\rangle$. Then $\tau_N=\{0_N,W_1,1_N\}$ and $\eta_N=\{0_N,W_2,1_N\}$ are NTSs on X and Y respectively. Define a bijective mapping $g_N:(X,\tau_N)\to(Y,\eta_N)$ by $f(a)=u$ and $f(b)=v$. Then g_N^{-1}(V) is not an NGCS in X. Therefore g_N is not an Neutrosophic G continuous mapping. Therefore the bijective mapping g_N is not an Neutrosophic Generalized Homeomorphism. But g_N is an N $\alpha\pi g$ continuous mapping and g_N^{-1} is also an N $\alpha\pi g$ continuous mapping. Therefore the bijective mapping g_N is an N $\alpha\pi g$ homeomorphism.

Remark 3.10:

An N $\alpha\pi g$ homeomorphism is independent of an NGS homeomorphism.

Example 3.11:

Let $X=\{a,b\}$, $Y=\{u,v\}$ and $W_1=\langle x,(0.7,0.8,0.3),(0.6,0.2,0.3)\rangle$ $W_2=\langle y,(0.3,0.4,0.6),(0.5,0.7,0.4)\rangle$. Then $\tau_N=\{0_N,W_1,1_N\}$ and $\eta_N=\{0_N,W_2,1_N\}$ are NTSs on X and Y respectively. Define a bijective mapping $g_N:(X,\tau_N)\to(Y,\eta_N)$ by $f(a)=u$ and $f(b)=v$. Then g_N^{-1}(V) is not an NGSCS in X. Therefore g_N is not an Neutrosophic GS continuous mapping. Therefore the bijective mapping g_N is not an Neutrosophic Generalized Semi Homeomorphism. But g_N is an N $\alpha\pi g$ continuous mapping and g_N^{-1} is also an N $\alpha\pi g$ continuous mapping. Therefore the bijective mapping g_N is an N $\alpha\pi g$ homeomorphism.

Example 3.12:

Let $X=\{a,b\}$, $Y=\{u,v\}$ and $W_1=\langle x,(0.8,0.5,0.5),(0.7,0.5,0.3)\rangle$ $W_2=\langle y,(0.5,0.5,0.6),(0.4,0.5,0.4)\rangle$. Then $\tau_N=\{0_N,W_1,1_N\}$ and $\eta_N=\{0_N,W_2,1_N\}$ are NTSs on X and Y respectively. Define a bijective mapping $g_N:(X,\tau_N)\to(Y,\eta_N)$ by $f(a)=u$ and $f(b)=v$. Then g_N^{-1}(V) is not an N $\alpha\pi g$ CS in X. Therefore g_N is not an an N $\alpha\pi g$ continuous mapping. Therefore the bijective mapping g_N is not an an N $\alpha\pi g$ Homeomorphism. But g_N is an NGS continuous mapping and g_N^{-1} is also an NGS continuous mapping. Therefore the bijective mapping g_N is an NGS homeomorphism.

Remark 3.13:

An N $\alpha\pi g$ homeomorphism is independent of an NGSP homeomorphism.

Example 3.14:

Let $X=\{a,b\}$, $Y=\{u,v\}$ and $W_1=\langle x,(0.3,0.3,0.7),(0.8,0.5,0.1)\rangle$ $W_2=\langle y,(0.7,0.7,0.2),(0.9,0.8,0.6)\rangle$. Then $\tau_N=\{0_N,W_1,1_N\}$ and $\eta_N=\{0_N,W_2,1_N\}$ are NTSs on X and Y respectively. Define a bijective mapping $g_N:(X,\tau_N)\to(Y,\eta_N)$ by $f(a)=u$ and $f(b)=v$. Consider an NS $K=\langle x,(0.4,0.4,0.3),(0.9,0.6,0.1)\rangle$ is an NCS in Y. Then $g_N^{-1}(K)$ is not an NGSPCS in X. Therefore g_N is not an Neutrosophic GSP continuous mapping. Therefore the bijective mapping g_N is not an Neutrosophic Generalized Semi Pre Homeomorphism. But g_N is an N $\alpha\pi g$ continuous mapping and g_N^{-1} is also an N $\alpha\pi g$ continuous mapping. Therefore the bijective mapping g_N is an N $\alpha\pi g$ homeomorphism.

Example 3.15:

Let $X = \{a,b\}$, $Y = \{u,v\}$ and $W_1 = \langle x,(0.9,0.5,0.5),(0.8,0.5,0.3)\rangle$ $W_2 = \langle y,(0.5,0.5,0.7),(0.3,0.5,0.3)\rangle$. Then $\tau_N = \{0_N, W_1, 1_N\}$ and $\eta_N = \{0_N, W_2, 1_N\}$ are NTSs on X and Y respectively. Define a bijective mapping $g_N : (X,\tau_N) \to (Y,\eta_N)$ by $f(a) = u$ and $f(b) = v$. Then ${g_N}^{-1}(V)$ is not an N$\alpha\pi g$ CS in X. Therefore g_N is not an an N$\alpha\pi g$ continuous mapping. Therefore the bijective mapping g_N is not an an N$\alpha\pi g$ Homeomorphism. But g_N is an NGSP continuous mapping and ${g_N}^{-1}$ is also an NGSP continuous mapping. Therefore the bijective mapping g_N is an NGSP homeomorphism.

Theorem 3.16:

Let $g_N : (X,\tau_N) \to (Y,\eta_N)$ be an N$\alpha\pi g$ homeomorphism if X and Y are N$\alpha\pi$ a$T_{1/2}$ space.

Proof:

Let V be an NCS in Y. Since g_N is an N$\alpha\pi g$ homeomorphism, g_N is an N$\alpha\pi g$ continuous mapping. Therefore ${g_N}^{-1}(V)$ is an N$\alpha\pi g$ CS in X. Since X is an N$\alpha\pi$ a$T_{1/2}$ space, ${g_N}^{-1}(V)$ is an NCS in X. Hence g_N is an Neutrosophic continuous mapping. Let D be an NCS in X. Since g_N is an N$\alpha\pi g$ homeomorphism, ${g_N}^{-1}$ is an N$\alpha\pi g$ continuous mapping. Therefore $({g_N}^{-1})^{-1}(D) = g_N(D)$ is an N$\alpha\pi g$ CS in X. Since X is an N$\alpha\pi$ a$T_{1/2}$ space, $g_N(D)$ is an NCS in X. Hence ${g_N}^{-1}$ is an Neutrosophic Continuous mapping. Therefore g_N is an Neutrosophic homeomorphism.

Neutrosophic $i-\alpha\pi g$ Homeomorphisms

In this section, we introduce Neutrosophic $i-\alpha\pi g$ homeomorphisms and study some of their properties.

Definition 4.1:

A bijective mapping $g_N : (X,\tau_N) \to (Y,\eta_N)$ is called an Neutrosophic $i-\alpha\pi g$ homeomorphism (N $i-\alpha\pi g$ homeomorphism in short) if g_N and ${g_N}^{-1}$ are N$i-\alpha\pi g$ irresolute mappings.

Example 4.2:

Let $X = \{a,b\}$, $Y = \{u,v\}$ and $W_1 = \langle x,(0.9,0.5,0.5),(0.8,0.5,0.3)\rangle$ $W_2 = \langle y,(0.5,0.5,0.7),(0.3,0.5,0.3)\rangle$. Then $\tau_N = \{0_N, W_1, 1_N\}$ and $\eta_N = \{0_N, W_2, 1_N\}$ are NTSs on X and Y respectively. Define a bijective mapping $g_N : (X,\tau_N) \to (Y,\eta_N)$ by $f(a) = u$ and $f(b) = v$. Then g_N is an N$\alpha\pi g$ irresolute mapping and ${g_N}^{-1}$ is also an N $\alpha\pi g$ irresolute mapping. Therefore the bijective mapping g_N is an N$i-\alpha\pi g$ homeomorphism.

Theorem 4.3:

Every Neutrosophic N$i-\alpha\pi g$ Homeomorphism is an N$\alpha\pi g$ homeomorphism but not conversely.

Proof:

Let $g_N : (X,\tau_N) \to (Y,\eta_N)$ be an N$i-\alpha\pi g$ homeomorphism. Let V be an NCS in Y. This implies V is an N $\alpha\pi g$ CS in V. By hypothesis, $({g_N}^{-1})^{-1}(V) = g_N(V)$ N$\alpha\pi g$ CS in X. Hence ${g_N}^{-1}$ is an N$\alpha\pi g$ continuous mappings. This implies g_N N$\alpha\pi g$ homeomorphism.

Example 4.4:

Let $X = \{a,b\}$, $Y = \{u,v\}$ and $W_1 = \langle x,(0.4,0.5,0.6),(0.3,0.5,0.7)\rangle$ $W_2 = \langle y,(0.7,0.2,0.1),(0.8,0.9,0.2)\rangle$. Then $\tau_N = \{0_N, W_1, 1_N\}$ and $\eta_N = \{0_N, W_2, 1_N\}$ are NTSs on X and Y respectively. Define a bijective mapping $g_N : (X,\tau_N) \to (Y,\eta_N)$ by $f(a) = u$ and $f(b) = v$. Then $g_N^{-1}(V)$ is not an N$\alpha\pi g$ CS in X. Therefore g_N is not an N$\alpha\pi g$ irresolute mapping. Therefore the bijective mapping g_N is not an N$i-\alpha\pi g$ Homeomorphism. But g_N is an N$\alpha\pi g$ continuous mapping and g_N^{-1} is also an N$\alpha\pi g$ continuous mapping. Therefore the bijective mapping g_N is an N$\alpha\pi g$ homeomorphism.

Definition 4.5:

Let V be an NS in an NTS (X,τ_N). Then $\alpha\pi gcl(V)$ is defined as

$$\alpha\pi gcl(V) = \cap\{D \mid D \text{ is an N}\alpha\pi g \text{ CS in } X \text{ and } V \subseteq D\}.$$

Theorem 4.6:

If the mapping $g_N : (X,\tau_N) \to (Y,\eta_N)$ is an N$i-\alpha\pi g$ homeomorphism, then $\alpha\pi gcl(g_N^{-1}(D)) \subseteq g_N^{-1}(\alpha cl(D))$ for every NS D in Y.

Proof:

Let D be an NS in Y. Then $\alpha cl(D)$ is an $N\alpha CS$ in Y. This implies $\alpha cl(D)$ is an N$\alpha\pi g$ CS in Y. Since g_N is an N$\alpha\pi g$ irresolute mapping, $g_N^{-1}(\alpha cl(D))$ is an N$\alpha\pi g$ CS in X. This implies $\alpha\pi gcl(g_N^{-1}(\alpha cl(D))) = g_N^{-1}(\alpha cl(D))$. Now $\alpha\pi gcl(g_N^{-1}(D)) \subseteq \alpha\pi gcl(g_N^{-1}(\alpha cl(D))) = g_N^{-1}(\alpha cl(B))$. Hence $\alpha\pi gcl(g_N^{-1}(D)) \subseteq g_N^{-1}(\alpha cl(D))$ for every NS D in Y.

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