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Decomposition and Characterization of $V_{\mathcal{G}} - \mathrm{M_p}$ continuous functions in Grill V − space

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Abstract:

The proposed paper introduces a notations $V_{\mathcal{G}} - \mathcal{M}_p$ − continuous and $V_{\mathcal{G}} - \mathcal{M}_p$ *open* ($V_{\mathcal{G}} - \mathcal{M}_p$ *closed function*) Function by using $V_{\mathcal{G}} - \mathcal{M}_p$ *open sets and investigate some their* Properties in *Grill V − space*.

Keywords: $V_{\mathcal{G}} - \mathcal{M}_p$ − continuous, $V_{\mathcal{G}} - \mathcal{M}_p$ − open function $V_{\mathcal{G}} - \mathcal{M}_p$ *open*, $V_{\mathcal{G}} - \mathcal{M}_p\ O(X)$, $V_{\mathcal{G}} - open$, $V_{\mathcal{G}} - pre\ open$.

1. Introduction:

Choquet [1] introduced the notion of grill. After that, Roy [2] introduced the notion of grill topological space. In 2022, [3] introduced the concept of $V - open$ set and $V - Sapce$ and prove that this space represents a topological space under the certain conditions. In 2023, Esmaeel [4] introduce grill into this $V - Sapce$ as grill $V - Sapce$. In 2024, Manikandan N[5] introduce the new kind open set in Grill V- Space named as $V_G - M_p$ open set. In this article we propose new kind of continuous function called $V_G - M_p$ continuous by using $V_G - M_p$ open sets and investigated some of their properties.

2. Preliminaries

Definition 2.1

Let X represent a set that is non-empty and let $\{\tau_J\}_{J\in \mathrm{N}}, J \geq 2$ be any topologies on X and let the family $VO_X = \{\aleph \subseteq X: \aleph = \Phi\ or\ \exists \mathrm{T} \in \cap_{J\in\mathrm{N}}\ \tau_J\ such\ that\ \Phi \neq \mathrm{T} \subseteq \aleph\}$ satisfying the following axioms:

1. $X, \Phi \in VO_X$.
2. $\cup_{i\in\mathrm{N}} \mathrm{N}_i \in VO_X\ \forall \{\mathrm{N}_i\}_{i\in\mathrm{N}} \in VO_X$
3. $\cap_{i=1}^{n} \mathrm{N}_i \in VO_X\ \forall \{\mathrm{N}_i\}_{i=1}^{n} \in VO_X$

Then (X, VO_X) is called to be $V - Sapce$ and the elements of VO_X are called $V - open$ Sets and the complement of $V - open$ set is called $V - Closed$.

Definition 2.2

A non-empty family $\mathcal{G}$ of sets of $V - Space\ (X, VO_X)$ is named a grill on X if satisfies the following axioms

1. $\Phi \notin \mathcal{G}$.
2. $\mathbb{M} \in \mathcal{G}$ and $\mathbb{M} \subseteq \mathbb{P} \Rightarrow \mathbb{P} \in \mathcal{G}$.
3. $\mathbb{M} \notin \mathcal{G}\ and\ \mathbb{P} \notin \mathcal{G} \Rightarrow \mathbb{M} \cup \mathbb{P} \notin \mathcal{G}$.

Any $V-Space$ (X, VO_X) with a grill $\mathcal{G}$ on X is named grill $V-Space$ and is symbolize by $(X, VO_X, \mathcal{G})$.

Definition 2.3

Let $(X, VO_X, \mathcal{G})$ *be a grill* $V-Space$ *and let* $\aleph$ *be a subset of* X. *Then* $\aleph$ *is said to be* $V_{\mathcal{G}}-\mathcal{M}_p$ *open set if there exist a* $\mathrm{K} \in V_{\mathcal{G}}-PO(X)$ *such that* $\mathrm{K} \subseteq \aleph \subseteq \Psi(\mathrm{K})$. A set $\aleph$ Of X is $V_{\mathcal{G}}-\mathcal{M}_p$ closed if its complement $X-\aleph$ *is* $V_{\mathcal{G}}-\mathcal{M}_p$ *open*.

3. $V_{\mathcal{G}}-\mathcal{M}_p$ Continuous

Definition 3.1

A function $f:(X, VO_X, \mathcal{G}) \rightarrow (Y, \sigma)$ *is said to be* $V_{\mathcal{G}}-\mathcal{M}_p$ continuous if $f^{-1}(\mathrm{W}) \in V_{\mathcal{G}}-\mathcal{M}_p\ \mathrm{O(X)}$ *for all* $W \in PO(Y)$

Example 3.1:

Let $X=\{‡_1, ‡_2, ‡_3, ‡_4\}$, *and* $Y=\{a, b, c, d\}$: $VO_X=\{X, \Phi, \{‡_1, ‡_2\}, \{‡_3, ‡_4\}\}, \sigma = \{\Phi, Y, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$ and grill $\mathcal{G}=\{\{‡_1, ‡_2, ‡_3\}, X\}$. *Then* $V_{\mathcal{G}}-\mathcal{M}_p\ \mathrm{O(X)}=P(X)$ *and* $PO(Y) = \{\Phi, Y, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{c, d\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$.

Define $f:(X, VO_X, \mathcal{G}) \rightarrow (Y, \sigma)$ *by* $f(‡_1)=b, f(‡_2)=a, f(‡_3)=d, f(‡_4)=c$. *Then inverse image of* every preopen sets in Y *is* $V_{\mathcal{G}}-\mathcal{M}_p$ open in X. Hence f is $V_{\mathcal{G}}-\mathcal{M}_p$ continuous .

Theorem 3.1: *For a function* $f:(X, VO_X, \mathcal{G}) \rightarrow (Y, \sigma)$, *the follwoing are equivalent*:

(i) f ***is*** $V_{\mathcal{G}}-\mathcal{M}_p$ continuous

(ii) *For each* $W \in PC(Y), f^{-1}(\mathrm{W}) \in V_{\mathcal{G}}-\mathcal{M}_p\ \mathrm{C(X)}$

(iii) *For each* $x \in X$ *and each* $W \in PO(Y)$ *containg* $f(x)$, *there exists a* $U \in V_{\mathcal{G}}-\mathcal{M}_p\ \mathrm{O(X)}$ *containg* x *such that* $f(U) \subseteq W$

Proof: $(i) \Leftrightarrow (ii)$ *its obvious*.

$(\boldsymbol{i}) \Rightarrow (\boldsymbol{iii})$: *Let* $W \in \mathrm{V}_{\mathcal{G}}-\mathrm{PO(Y)}$ *and* $f(x) \in W$. *Then by* $(i) f^{-1}(\mathrm{W}) \in V_{\mathcal{G}}-\mathcal{M}_p\ \mathrm{O(X)}$ *containg* x.

Taking $f^{-1}(\mathrm{W})=\mathrm{U}$, we have that $\mathrm{x} \in \mathrm{U}$ and $f(U) \subseteq W$.

$(\boldsymbol{iii}) \Rightarrow (\boldsymbol{i})$: *Let* $W \in \mathrm{V}_{\mathcal{G}}-\mathrm{PO(Y)}$ *and* $x \in f^{-1}(\mathrm{W})$. *Then* $f(x) \in W \in PO(Y)$ *and hence by* (iii), *there exist a* $U \in V_{\mathcal{G}}-\mathcal{M}_p\ \mathrm{O(X)}$ containg x such that $f(U) \subseteq W$. *Then, we obtain* $\boldsymbol{x} \in \boldsymbol{U} \in \Psi(\mathrm{V}_{\mathcal{G}}-\mathrm{Pint(U)} \subseteq \Psi(\mathrm{V}_{\mathcal{G}}-\mathrm{Pint}(\mathrm{f}^{-1}(\mathrm{W})))$. This shows that $f^{-1}(\mathrm{W}) \subseteq \Psi(\mathrm{Pint}(\mathrm{f}^{-1}(\mathrm{W})))$.

Hence f is $\mathrm{V}_{\mathcal{G}}-\mathcal{M}_\mathrm{p}$ continuous.

Theorem 3.2: A function $\mathrm{h}:(\mathrm{X}, \mathrm{VO_X}, \mathcal{G}) \rightarrow (\mathrm{Y}, \sigma)$ is $\mathrm{V}_{\mathcal{G}}-\mathcal{M}_\mathrm{p}$ continuous if and only if the graph function $\mathrm{g}: \mathrm{X} \rightarrow \mathrm{X} \times \mathrm{Y}$, defined by $\mathrm{g(x)}=(\mathrm{x}, \mathrm{h(x)})$ for each $\mathrm{x} \in \mathrm{X}$, is $\mathrm{V}_{\mathcal{G}}-\mathcal{M}_\mathrm{p}$ continuous.

Proof: Assume that f is $V_{\mathcal{G}} - \mathcal{M}_p$ continuous. Let $x \in X$ and $V \in V_{\mathcal{G}} - PO(X \times Y)$ containing g(x). Then there exist a $W \in V_{\mathcal{G}} - PO(X)$ and $U \in V_{\mathcal{G}} - PO(Y)$ such that $g(x) = (x, f(x)) \in W \times U \subseteq V$. Since f is $V_{\mathcal{G}} - \mathcal{M}_p$ continuous, There exist a $H \in V_{\mathcal{G}} - \mathcal{M}_p\ O(X)$ containing x such that $f(H) \subseteq U$. Then $H \cap W \in V_{\mathcal{G}} - \mathcal{M}_p\ O(X)$ and $g(H \cap W) \in W \times U \subseteq V$. This shows that g is $V_{\mathcal{G}} - \mathcal{M}_p$ continuous.

Conversely, suppose g is $V_{\mathcal{G}} - \mathcal{M}_p$ continuous.Let $x \in X$ and $V \in V_{\mathcal{G}} - PO(Y)$ containing f(x). Then $X \times V \in V_{\mathcal{G}} - PO(X \times Y)$ and by $V_{\mathcal{G}} - \mathcal{M}_p$ continuity of g, there exist a $U \in V_{\mathcal{G}} - \mathcal{M}_p$ continuous
containing x such that $g(U) \subseteq X \times V$. Thus we have that $f(U) \subseteq V$ and hence f is $V_{\mathcal{G}} - \mathcal{M}_p$ continuous .

Remark 3.1: The concept of $V_{\mathcal{G}} -$ semicontinuous and $V_{\mathcal{G}} - \mathcal{M}_p$ continuous are independent.

In example 3.1, we have that $V_{\mathcal{G}} - SO(X) = \{ X, \Phi, \{\delta_1, \delta_2\}, \{\delta_3, \delta_4\}\}$ and the function f is $V_{\mathcal{G}} - \mathcal{M}_p$ continuous. Also $f^{-1}(\{a, b, c\}) = \{\delta_1, \delta_2, \delta_4\}$ is not $V_{\mathcal{G}} -$ semi open in X for
the open set $\{a, b, c\}$ of Y. Hence f is not $V_{\mathcal{G}} -$ semi continuous.

4. $V_{\mathcal{G}} - \mathcal{M}_p$ Open Function

Definition 4.1: Let (X, τ) be a topological space and $(Y, VO_X, \mathcal{G})$ be a grill V-Space. A function $f: (X, \tau) \to (Y, VO_X, \mathcal{G})$ is said to be $V_{\mathcal{G}} - \mathcal{M}_p$ Open $(\text{resp}V_{\mathcal{G}} - \mathcal{M}_p$ Closed$)$if for each $U \epsilon V_{\mathcal{G}} PO(X)$ $(\text{resp} U \epsilon V_{\mathcal{G}} - PC(X))$, f(U) is $V_{\mathcal{G}} - \mathcal{M}_p$ Open$(\text{resp } V_{\mathcal{G}} - \mathcal{M}_p$ Closed$)$ in $(Y, VO_X, \mathcal{G})$.

Theorem 4.1: A function $f: (X, \tau) \to (Y, VO_X, \mathcal{G})$ is $V_{\mathcal{G}} - \mathcal{M}_p$ Open if and only if for each $x \in X$
and each $V_{\mathcal{G}} -$ preneighbourhood U of x, there exist a $W \in V_{\mathcal{G}} - \mathcal{M}_p\ O(Y)$ such that $f(x) \in W \subseteq f(U)$.

Proof: Suppose that f is a $V_{\mathcal{G}} - \mathcal{M}_p$ open function and let $x \in X$. Also let U be any $V_{\mathcal{G}} -$ preneighbourhood
of x. Then there exist $G \in V_{\mathcal{G}} - PO(X)$ such that $x \in G \subseteq U$. Since f is $V_{\mathcal{G}}\_\mathcal{M}_p$ Open, $f(G) = W \in V_{\mathcal{G}} - \mathcal{M}_p\ O(Y)$ and $f(x) \in W \subseteq f(U)$. Conversely, suppose that $U \in V_{\mathcal{G}} - PO(X)$. Then for each $x \in U$,
there exist a $W_x \in V_{\mathcal{G}} - \mathcal{M}_p\ O(X)$ such that $f(x) \in W_x \subseteq f(U)$. Thus $f(U) = \cup \{W_x : x \in U\}$ and hence $f(U) \in V_{\mathcal{G}} - \mathcal{M}_p\ O(X)$. This shows that $V_{\mathcal{G}} - \mathcal{M}_p$ Open .

Theorem 4.2: Let $f: (X, \tau) \to (Y, VO_X, \mathcal{G})$ be a $V_{\mathcal{G}} - \mathcal{M}_p$ open function. if $W \subseteq Y$ and $F \in V_{\mathcal{G}} - PC(X)$
containg $f^{-1}(W)$, then there exist a $H \in V_{\mathcal{G}} - \mathcal{M}_p\ O(Y)$ containing W such that $f^{-1}(H) \subseteq F$.

Proof: Suppose that f is $V_{\mathcal{G}} - \mathcal{M}_p$ open. Let $W \subseteq Y$ and $F \in V_{\mathcal{G}} - PC(X)$ containing $f^{-1}(V)$. Then $X - F \in V_{\mathcal{G}} - PO(X)$ and by $V_{\mathcal{G}} - \mathcal{M}_p$ openness of f, $f(X - F) \in V_{\mathcal{G}} -$

$\mathcal{M}_p$ O(X). Thus $H = Y - f(X - F) \in V_{\mathcal{G}} - \mathcal{M}_p C(Y)$ consequently $f^{-1}(W) \subseteq F$ implies that $W \subseteq H$. Further, we obtain that $f^{-1}(H) \subseteq F$.

Theorem 4.3: For any bijection $f: (X, \tau) \to (Y, VO_X, \mathcal{G})$, the following are equivalent

(i) $f^{-1}: (Y, VO_X, \mathcal{G}) \to (X, \tau)$ is $V_{\mathcal{G}} - \mathcal{M}_p$ continuous;

(ii) f is $V_{\mathcal{G}} - \mathcal{M}_p$ Open;

(iii) f is $V_{\mathcal{G}} - \mathcal{M}_p$ Closed.

Proof: it is obvious.

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