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# PAIRWISE $g_f$ −CONTINUOUS MAPS ON $g_f$ −BITOPOLOGICAL SPACES

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**ABSTRACT**. In this paper we defined and characterized the concept of generalized fuzzy bi-topological space ($g_f$ −bi-topological space) and obtained some significant results in this context. Further, we study the concept of continuity called Pairwise $g_f -$ continuous maps in $g_f$ −bi-topological spaces and established the several relationships by making the use of some examples.

**KEYWORDS:** Fuzzy bi-topological space, $g_f$ −bi-topological space, pairwise $g_f -$ continuous maps in $g_f$ −bi-topological space

## 1. INTRODUCTION

One of the earliest branches of mathematics which applied fuzzy set theory systematically is General Topology. The ideas, notions and methods of fuzzy set theory are synthesized with those of general topology to introduce fuzzy topology as a new member in the branch of mathematics. Fuzzy topology can deepen the understanding of basic structure of classical mathematics, it is a generalization of topology in classical mathematics; it has its own marked characteristics. Zadeh [21] has introduced the notion of fuzzy set which is a significant notion in the theory of fuzzy mathematics. Chang [6] has introduced the concept of fuzzy topological space as a generalization of topological space and Kandil [12] introduced fuzzy bi-topological spaces in 1989.
Azad [1] has introduced the concept of fuzzy semi-open sets in fuzzy topological spaces. Bin Shahana [4-5] has introduced the concept of fuzzy pre-open sets and fuzzy α-open sets in fuzzy topological space. Thakur [18] has introduced the concept of fuzzy semi pre-open sets in fuzzy topological spaces. Beceren [2] introduced and studied the concept of strongly α-continuous functions, strong semi-continuity and fuzzy pre-continuity and investigate various characterizations. Further the author verified that fuzzy strongly α-continuous map is the stronger form of fuzzy α-continuous map. Csaszar [7] introduced the notions of generalized topological spaces. He also introduced the notions of continuous functions and associated interior and closure operators on generalized neighbourhood systems and generalized topological spaces. Palani Cheety [13] introduced the concept of generalized fuzzy topology and investigates various properties.

ORGANIZATION: The rest of the paper structured as follows: Some require basic definitions, concepts of fuzzy bi-topological space and notations are discussed in Section 2. The section 3 has been headed by the concept of $\mathrm{g_F}$ −bi-topological space in which we discuss several concepts relayed to $\mathrm{g_F}$ −bi-topological space and established the several relationships by making the use of some examples. Section 4 has been headed by the concept of pairwise $\mathrm{g_f}$ − continuous maps in which we discuss the concept of continuity in $\mathrm{g_f}$ −bi-topological space and established the several relationships by making the use of some counter examples. Finally, Section 5 concludes this paper.

## 2. PRELIMINARIES

Definition 2.1: Let $(\mathrm{X}, \tau_1, \tau_2)$ consisting of a universal set X with the fuzzy topologies "$\tau_1$" and "$\tau_2$" on X is called fuzzy bi-topological space

Definition 2.2: A fuzzy set A of fuzzy bi-topological space $(\mathrm{X}, \tau_1, \tau_2)$ is called fuzzy $(\mathrm{i}, \mathrm{j}) - \mathrm{semi} - \mathrm{open\ set}$ if $\mathrm{A} \subseteq \tau_\mathrm{j} - \mathrm{cl}(\tau_\mathrm{i} - \mathrm{int}(\mathrm{A}))$. In a fuzzy bi-topological space $(\mathrm{X}, \tau_1, \tau_2)$, every fuzzy $\tau_\mathrm{i} - \mathrm{open\ set}$ $(i = 1,2)$ is fuzzy $(\mathrm{i}, \mathrm{j}) - \mathrm{semi} - \mathrm{open\ set}$ but not converse. A fuzzy set A of fuzzy bi-topological space $(\mathrm{X}, \tau_1, \tau_2)$ is called fuzzy $(\mathrm{i}, \mathrm{j}) - \mathrm{semi} - \mathrm{closed\ set}$ if $\mathrm{A^c} = 1 - \mathrm{A}$ is fuzzy $(\mathrm{i}, \mathrm{j}) - \mathrm{semi} - \mathrm{open\ set}$

Definition 2.3: A fuzzy set A of fuzzy bi-topological space $(\mathrm{X}, \tau_1, \tau_2)$ is called fuzzy $(\mathrm{i}, \mathrm{j}) - \mathrm{pre} - \mathrm{open\ set}$ if $\mathrm{A} \subseteq \tau_\mathrm{i} - \mathrm{int}(\tau_\mathrm{j} - \mathrm{cl}(\mathrm{A}))$. A fuzzy set A of fuzzy bi-topological space $(\mathrm{X}, \tau_1, \tau_2)$ is called fuzzy $(\mathrm{i}, \mathrm{j}) - \mathrm{pre} - \mathrm{closed\ set}$ if $\mathrm{A^c} = 1 - \mathrm{A}$ is fuzzy $(\mathrm{i}, \mathrm{j}) - \mathrm{pre} - \mathrm{open\ set}$.

Definition 2.4: A fuzzy set A of fuzzy bi-topological space $(\mathrm{X}, \tau_1, \tau_2)$ is called fuzzy $(\mathrm{i}, \mathrm{j}) - \beta - \mathrm{open\ set}$ if $\mathrm{A} \subseteq \tau_\mathrm{j} - \mathrm{cl}(\tau_\mathrm{i} - \mathrm{int}\left(\tau_\mathrm{j} - \mathrm{cl}(\mathrm{A})\right))$. A fuzzy set A of fuzzy bi-topological space $(\mathrm{X}, \tau_1, \tau_2)$ is called fuzzy $(\mathrm{i}, \mathrm{j}) - \beta - \mathrm{closed\ set}$ if $\mathrm{A^c} = 1 - \mathrm{A}$ isfuzzy $(\mathrm{i}, \mathrm{j}) - \beta - \mathrm{open\ set}$.

Definition 2.5: A fuzzy set A of fuzzy bi-topological space $(\mathrm{X}, \tau_1, \tau_2)$ is called fuzzy $(\mathrm{i}, \mathrm{j}) - \alpha - \mathrm{open\ set}$ if $\mathrm{A} \subseteq \tau_\mathrm{j} - \mathrm{int}(\tau_\mathrm{i} - \mathrm{cl}\left(\tau_\mathrm{j} - \mathrm{int}(\mathrm{A})\right))$. A fuzzy set A of fuzzy bi-topological space $(\mathrm{X}, \tau_1, \tau_2)$ is called fuzzy $(\mathrm{i}, \mathrm{j}) - \alpha - \mathrm{closed\ set}$ if $\mathrm{A^c} = 1 - \mathrm{A}$ is fuzzy $(\mathrm{i}, \mathrm{j}) - \alpha - \mathrm{open\ set}$.

Definition 2.6: A fuzzy set A of fuzzy bi-topological space $(\mathrm{X}, \tau_1, \tau_2)$ is called

Fuzzy $(\mathrm{i}, \mathrm{j}) - \mathrm{regular} - \mathrm{open\ set}$ if $\tau_\mathrm{j} - \mathrm{int}\left(\tau_\mathrm{i} - \mathrm{cl}(\mathrm{A})\right) = \mathrm{A}$

Fuzzy $(\mathrm{i}, \mathrm{j}) - \mathrm{regular} - \mathrm{closed\ set}$ if $\mathrm{t_j} - \mathrm{cl}\left(\mathrm{t_i} - \mathrm{int}(\mathrm{A})\right) = \mathrm{A}$

## 3. $\mathbf{g_f}$ −BI-TOPOLOGICAL SPACES

Definition 3.1: Let $(\mathrm{X}, \mathrm{t_1}, \mathrm{t_2})$ consisting of a universal set X with the $\mathrm{g_f}$ −topologies "$\mathrm{t_1}$" and "$\mathrm{t_2}$" on X is called $\mathrm{g_f}$ −bi-topological space

Example 3.1: Let $\mathrm{X} = \{\mathrm{x_1}, \mathrm{x_2}\}$ and we consider fuzzy sets $\mathrm{A} = \left\{\left(\frac{\mathrm{x_1}}{0.3}\right), \left(\frac{\mathrm{x_2}}{0.6}\right)\right\}$, $\mathrm{B} = \left\{\left(\frac{\mathrm{x_1}}{0.5}\right), \left(\frac{\mathrm{x_2}}{0.4}\right)\right\}$, $\mathrm{C} = \left\{\left(\frac{\mathrm{x_1}}{0.5}\right), \left(\frac{\mathrm{x_2}}{0.6}\right)\right\}$, $\mathrm{D} = \left\{\left(\frac{\mathrm{x_1}}{0.2}\right), \left(\frac{\mathrm{x_2}}{0.5}\right)\right\}$, $\mathrm{E} = \left\{\left(\frac{\mathrm{x_1}}{0.5}\right), \left(\frac{\mathrm{x_2}}{0.4}\right)\right\}$ and $\mathrm{F} = \left\{\left(\frac{\mathrm{x_1}}{0.6}\right), \left(\frac{\mathrm{x_2}}{0.5}\right)\right\}$ on X. Then clearly $\mathrm{t_1} = \{0, \mathrm{A}, \mathrm{B}, \mathrm{C}, 1\}$ and $\mathrm{t_2} = \{0, \mathrm{D}, \mathrm{E}, \mathrm{F}, 1\}$ are $\mathrm{g_f} - \mathrm{topologies}$ on X. Then $(\mathrm{X}, \mathrm{t_1}, \mathrm{t_2})$ is a $\mathrm{g_f}$ bi-topological space

Definition 3.2: A fuzzy set A of $g_f$ −bi-topological space $(X, t_1, t_2)$ is called $g_f - (i, j) -$ semi − open set if $A \subseteq t_j - Cl_t(t_i - Int_t(A))$

Example 3.2: In Example 3.1, consider, $P = \left\{\left(\frac{x_1}{0.7}\right), \left(\frac{x_2}{0.8}\right)\right\}$ be a fuzzy set. Clearly $P \subseteq t_2 - Cl_t\left(t_1 - Int_t(P)\right)$ and hence P is $g_f - (1,2)$ −semi open set. Again $P \subseteq t_1 - Cl_t\left(t_2 - Int_t(P)\right)$, implies P is $g_f - (2,1)$ − semi open set. Hence P is $g_t - (I, j)$ − semi open set.

Remark 3.1: In a $g_f$ −bi-topological space $(X, t_1, t_2)$, every $g_f - t_i$ − open set $(i = 1,2)$ is $g_f - (i, j)$ − semi open set but not converse. In Example 3.2, P is $g_f - (i, j)$ − semi open set but not $g_f - t_i$ − open set $(i = 1,2)$.

Definition 3.3: A fuzzy set "A" of $g_f$ −bi-topological space $(X, t_1, t_2)$ is called $g_f - (i, j)$ − semi closed set if $A^c = 1 - A$ is $g_f - (i, j)$ − semi open set.

Definition 3.4: A fuzzy set A of $g_f$ −bi-topological space $(X, t_1, t_2)$ is called $g_f - (i, j) -$ pre − open set if $A \subseteq t_j - Int_t(t_i - Cl_t(A))$

Example 3.3: In Example 3.1, consider, $P = \left\{\left(\frac{x_1}{0.7}\right), \left(\frac{x_2}{0.8}\right)\right\}$ be a fuzzy set. Clearly $P \subseteq t_1 - Int_t(t_2 - Cl_t(P))$ and hence P is $g_f\ (1,2)$ − pre − open set in $(X, t_1, t_2)$

Definition 3.5: A fuzzy set A of $g_f$ −bi-topological space $(X, t_1, t_2)$ is called $g_f - (i, j) -$ pre − closed set if $A^c = 1 - A$ is $g_f - (i, j)$ − pre − open set. In Example 3.1, fuzzy set $Q = \left\{\left(\frac{x_1}{0.3}\right), \left(\frac{x_2}{0.2}\right)\right\}$ is $g_f - (1,2)$ − pre − closed set in $(X, t_1, t_2)$

Definition 3.6: A fuzzy set A of $g_f$ −bi-topological space $(X, t_1, t_2)$ is called $g_f - (i, j) - \beta -$ open set if $A \subseteq t_j - Cl_t(t_i - Int_t\left(t_j - Cl_t(A)\right))$. In Example 3.1, fuzzy set $P = \left\{\left(\frac{x_1}{0.7}\right), \left(\frac{x_2}{0.8}\right)\right\}$ is $g_t - (1,2) - \beta -$ open set in $(X, t_1, t_2)$

Definition 3.7: A fuzzy set A of $g_f$ −bi-topological space $(X, t_1, t_2)$ is called $g_f - (i, j) - \beta -$ closed set if $A^c = 1 - A$ is $g_t - (i, j) - \beta -$ open set. In Example 3.1, fuzzy set $Q = \left\{\left(\frac{x_1}{0.3}\right), \left(\frac{x_2}{0.2}\right)\right\}$ is $g_f - (1,2) - \beta -$ closed set in $(X, t_1, t_2)$

Remark 3.2: In a $g_f$ −bi-topological space$(X, t_1, t_2)$, every $g_f - (i, j)$ − pre − open set is $g_f - (i, j) - \beta -$ open set but not converse

Example 3.4: Let $X = \{x_1, x_2\}$ and we consider fuzzy sets $A = \left\{\left(\frac{x_1}{0.3}\right), \left(\frac{x_2}{0.6}\right)\right\}$, $B = \left\{\left(\frac{x_1}{0.5}\right), \left(\frac{x_2}{0.4}\right)\right\}$, $C = \left\{\left(\frac{x_1}{0.5}\right), \left(\frac{x_2}{0.6}\right)\right\}$, $D = \left\{\left(\frac{x_1}{0.2}\right), \left(\frac{x_2}{0.5}\right)\right\}$ , $E = \left\{\left(\frac{x_1}{0.6}\right), \left(\frac{x_2}{0.3}\right)\right\}$ and $F = \left\{\left(\frac{x_1}{0.6}\right), \left(\frac{x_2}{0.5}\right)\right\}$ on X. Then clearly $t_1 = \{0, A, B, C, 1\}$ and $t_2 = \{0, D, E, F, 1\}$ are $g_f$ −topologies on X. Then clearly the fuzzy set $P = \left\{\left(\frac{x_1}{0.8}\right), \left(\frac{x_2}{0.4}\right)\right\}$ is $g_f - (1,2) - \beta -$ open set but not $g_f - (1,2)$ − pre − open set in $(X, t_1, t_2)$

Definition 3.8: A fuzzy set A of $g_f$ −bi-topological space $(X, t_1, t_2)$ is called $g_f - (i, j) - \alpha -$ open set if $A \subseteq t_j - Int_t(t_i - Cl_t\left(t_j - Int_t(A)\right))$

Example 3.5: In Example 3.4, consider, $P = \left\{\left(\frac{x_1}{0.5}\right), \left(\frac{x_2}{0.5}\right)\right\}$ is $g_f - (1,2) - \alpha -$ open set in $(X, t_1, t_2)$

Remark 3.3: In a $g_f$ −bi-topological space$(X, t_1, t_2)$, every $g_f - (i,j) - \alpha - \text{open set}$ is $g_f - (i,j) - \text{semi} - \text{open set}$ but not converse. In Example 3.4, consider, $Q = \left\{\left(\frac{x_1}{0.4}\right), \left(\frac{x_2}{0.7}\right)\right\}$ is $- (1,2) - \text{semi} - \text{open set}$ but not $g_f - (1,2) - \alpha - \text{open set in } (X, t_1, t_2)$

Definition 3.9: A fuzzy set A of $g_f$ −bi-topological space $(X, t_1, t_2)$ is called

$g_f - (i,j) - \text{regular} - \text{open set if } t_j - \text{Int}_t\big(t_i - \text{Cl}_t(A)\big) = A$

$g_f - (i,j) - \text{regular} - \text{closed set if } t_j - \text{Cl}_t\big(t_i - \text{Int}_t(A)\big) = A$

Remark 3.4: In $g_f$ −bi-topological space $(X, t_1, t_2)$

Every $g_f - (i,j) - \text{regular} - \text{open set is } g_f - t_j - \text{open}$ but not converse

Every $g_f - (i,j) - \text{regular} - \text{closed set is } g_f - t_j - \text{closed}$ but not converse

## 4. PAIRWISE $\mathbf{g_f}$ −CONTINUOUS MAPS

Definition 4.1: A mapping f: $(X, t_1, t_2) \to (Y, g_1, g_2)$ is said to be pairwise $g_f - \text{continuous map}$ if f: $(X, t_1) \to (Y, g_1)$ and f: $(X, t_2) \to (Y, g_2)$ are $g_f - \text{continuous maps}$

Example 4.1: Let $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$.Consider fuzzy sets $A = \left\{\left(\frac{x_1}{0.3}\right), \left(\frac{x_2}{0.6}\right)\right\}$, $B = \left\{\left(\frac{x_1}{0.5}\right), \left(\frac{x_2}{0.4}\right)\right\}$, $C = \left\{\left(\frac{x_1}{0.5}\right), \left(\frac{x_2}{0.6}\right)\right\}$, $D = \left\{\left(\frac{x_1}{0.2}\right), \left(\frac{x_2}{0.5}\right)\right\}$, $E = \left\{\left(\frac{x_1}{0.6}\right), \left(\frac{x_2}{0.3}\right)\right\}$ and $F = \left\{\left(\frac{x_1}{0.6}\right), \left(\frac{x_2}{0.5}\right)\right\}$ on X. Again$P = \left\{\left(\frac{y_1}{0.3}\right), \left(\frac{y_2}{0.6}\right)\right\}$, $Q = \left\{\left(\frac{y_1}{0.5}\right), \left(\frac{y_2}{0.4}\right)\right\}$, $R = \left\{\left(\frac{y_1}{0.5}\right), \left(\frac{y_2}{0.6}\right)\right\}$, $S = \left\{\left(\frac{y_1}{0.2}\right), \left(\frac{y_2}{0.5}\right)\right\}$ , $T = \left\{\left(\frac{y_1}{0.6}\right), \left(\frac{y_2}{0.3}\right)\right\}$ and $U = \left\{\left(\frac{y_1}{0.6}\right), \left(\frac{y_2}{0.5}\right)\right\}$ on Y. Let $t_1 = \{0, A, B, C, 1\}$, $t_2 = \{0, D, E, F, 1\}$, $g_1 = \{0, P, Q, R, 1\}$ and $g_2 = \{0, S, T, U, 1\}$ be the $g_f - \text{topologies}$ defined on X and Y. Then we define a mapping f: $(X, t_1, t_2) \to (Y, g_1, g_2)$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$ which is pairwise $g_f - \text{continuous map}$

Definition 4.2: A mapping $f: (X, t_1, t_2) \to (Y, g_1, g_2)$ is said to be pairwise $g_f - \text{semi} - \text{continuous map}$ if inverse image of every $g_i - g_f - \text{open set}$ in Y is $g_f - (i,j) - \text{semi} - \text{open set in X}$

Example 4.2: In Example 4.1, the mapping f: $(X, t_1, t_2) \to (Y, g_1, g_2)$ defined as $f(x_1) = y_1$ and $f(x_2) = y_2$ is pairwise $g_f$ −semi-continuous map

Remark 4.1: In a $g_f$ −bi-topological space$(X, t_1, t_2)$every pairwise $g_f - \text{continuous map}$ is pairwise $g_f$ −semi-continuous map but not converse in general which is shown in Example 4.3

Example 4.3: Let $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$.Consider fuzzy sets $A = \left\{\left(\frac{x_1}{0.3}\right), \left(\frac{x_2}{0.6}\right)\right\}$, $B = \left\{\left(\frac{x_1}{0.5}\right), \left(\frac{x_2}{0.4}\right)\right\}$, $C = \left\{\left(\frac{x_1}{0.5}\right), \left(\frac{x_2}{0.6}\right)\right\}$, $D = \left\{\left(\frac{x_1}{0.2}\right), \left(\frac{x_2}{0.5}\right)\right\}$, $E = \left\{\left(\frac{x_1}{0.6}\right), \left(\frac{x_2}{0.3}\right)\right\}$ and $F = \left\{\left(\frac{x_1}{0.6}\right), \left(\frac{x_2}{0.5}\right)\right\}$ on X. Again$P = \left\{\left(\frac{y_1}{0.3}\right), \left(\frac{y_2}{0.6}\right)\right\}$, $Q = \left\{\left(\frac{y_1}{0.5}\right), \left(\frac{y_2}{0.4}\right)\right\}$, $R = \left\{\left(\frac{y_1}{0.5}\right), \left(\frac{y_2}{0.6}\right)\right\}$, $S = \left\{\left(\frac{y_1}{0.6}\right), \left(\frac{y_2}{0.6}\right)\right\}$ , $T = \left\{\left(\frac{y_1}{0.2}\right), \left(\frac{y_2}{0.5}\right)\right\}$ , $U = \left\{\left(\frac{y_1}{0.6}\right), \left(\frac{y_2}{0.3}\right)\right\}$ and $V = \left\{\left(\frac{y_1}{0.6}\right), \left(\frac{y_2}{0.5}\right)\right\}$ on Y. Let $t_1 = \{0, A, B, C, 1\}$, $t_2 = \{0, D, E, F, 1\}$, $g_1 = \{0, P, Q, R, S, 1\}$ and $g_2 = \{0, T, U, V, 1\}$ be the $g_f - \text{topologies}$ defined on X and Y. Then we define a mapping f: $(X, t_1, t_2) \to (Y, g_1, g_2)$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$ which is pairwise $g_f$ −semi-continuous map but not pairwise $g_f - \text{continuous map}$ because the set $S = \left\{\left(\frac{y_1}{0.6}\right), \left(\frac{y_2}{0.6}\right)\right\}$ is $g_1 - g_f - \text{open set in Y but not } t_i - g_f - \text{open set } (i = 1,2) \text{ in X.}$

Definition 4.3: A mapping $f:(X,t_1,t_2) \to (Y,g_1,g_2)$ is said to be pairwise $g_f -$ pre $-$ continuous map if inverse image of every $g_i - g_f -$ open set in Y is $g_f - (i,j) -$ pre $-$ open set in X

Example 4.4: In Example 4.1, the mapping $f:(X,t_1,t_2) \to (Y,g_1,g_2)$ defined as $f(x_1) = y_1$ and $f(x_2) = y_2$ is pairwise $g_f -$ pre $-$ continuous map

Remark 4.2: In a $g_f -$ bi-topological space$(X,t_1,t_2)$every pairwise $g_f -$ continuous map is pairwise $g_f -$ pre $-$ continuous map but not converse in general which is shown in Example 4.5

Example 4.5: In Example 4.3, consider $S = \left\{\left(\frac{y_1}{0.5}\right),\left(\frac{y_2}{0.3}\right)\right\}$ and $t_1 = \{0,A,B,C,1\}$, $t_2 = \{0,D,E,F,1\}$, $g_1 = \{0,P,Q,R,S,1\}$ and $g_2 = \{0,T,U,V,1\}$ be the $g_f -$ topologies defined on X and Y. Then we define a mapping $f:(X,t_1,t_2) \to (Y,g_1,g_2)$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$ which is pairwise $g_f -$ pre $-$ continuous map but not pairwise $g_f -$ continuous map because the set $S = \left\{\left(\frac{y_1}{0.5}\right),\left(\frac{y_2}{0.3}\right)\right\}$ is $g_1 - g_f -$ open set in Y but not $t_i - g_f -$ open set $(i = 1,2)$ in X.

Definition 4.4: A mapping $f:(X,t_1,t_2) \to (Y,g_1,g_2)$ is said to be pairwise $g_f -$ irresolute map if inverse image of every $g_f - (i,j) -$ semi $-$ open set in Y is $g_f - (i,j) -$ semi $-$ open set in X

Remark 4.3: In a $g_f -$bi-topological space$(X,t_1,t_2)$every pairwise $g_f -$ continuous map is pairwise $g_f -$ irresolute map but not converse in general which is shown in Example 4.6

Example 4.6: In Example 4.3, the mapping $f:(X,t_1,t_2) \to (Y,g_1,g_2)$is pairwise $g_f -$ irresolute map but not pairwise $g_f -$ continuous map because the set $S = \left\{\left(\frac{y_1}{0.6}\right),\left(\frac{y_2}{0.6}\right)\right\}$ is $g_1 - g_f -$ open set in Y but not $t_i - g_f -$ open set $(i = 1,2)$ in X.

Definition 4.5: A mapping $f:(X,t_1,t_2) \to (Y,g_1,g_2)$ is said to be pairwise $g_f - \alpha -$ continuous map if inverse image of every $g_i - g_f -$ open set in Y is $g_f - (i,j) - \alpha -$ open set in X

Remark 4.4: In a $g_f -$bi-topological space$(X,t_1,t_2)$every pairwise $g_f -$ continuous map is pairwise $g_f - \alpha -$ continuous map but not converse in general which is shown in Example 4.7

Example 4.7: In Example 4.3, the mapping $f:(X,t_1,t_2) \to (Y,g_1,g_2)$is pairwise $g_f - \alpha -$ continuous map but not pairwise $g_f -$ continuous map, the reason is same as in Example 4.3

Remark 4.5: In a $g_f -$bi-topological space $(X,t_1,t_2)$every pairwise $g_f - \alpha -$continuous map is pairwise $g_f -$ semi $-$ continuous map (pairwise $g_f -$ pre $-$ continuous map) but not converse in general which is shown in Example 4.8 and Example 4.9.

Example 4.8: In Example 4.3, consider $S = \left\{\left(\frac{y_1}{0.5}\right),\left(\frac{y_2}{0.5}\right)\right\}$ and $t_1 = \{0,A,B,C,1\}$, $t_2 = \{0,D,E,F,1\}$, $g_1 = \{0,P,Q,R,S,1\}$ and $g_2 = \{0,T,U,V,1\}$ be the $g_f -$ topologies defined on X and Y. Then we define a mapping $f:(X,t_1,t_2) \to (Y,g_1,g_2)$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$ which is pairwise $g_f -$ semi $-$ continuous map but not pairwise $g_f - \alpha -$ continuous map because the set $S = \left\{\left(\frac{y_1}{0.5}\right),\left(\frac{y_2}{0.5}\right)\right\}$ is $g_f - (i,j) -$ semi $-$ open set in X but not $g_f - (i,j) - \alpha -$ open set in X.

Example 4.9: Similarly, In Example 4.3, consider $S = \left\{\left(\frac{y_1}{0.5}\right), \left(\frac{y_2}{0.3}\right)\right\}$ and $t_1 = \{0, A, B, C, 1\}$, $t_2 = \{0, D, E, F, 1\}$, $g_1 = \{0, P, Q, R, S, 1\}$ and $g_2 = \{0, T, U, V, 1\}$ be the $g_f -$ topologies defined on X and Y. Then we define a mapping $f: (X, t_1, t_2) \to (Y, g_1, g_2)$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$ which is pairwise $g_f - \text{pre} - \text{continuous map}$ but not pairwise $g_f - \alpha - \text{continuous map}$ because the set $S = \left\{\left(\frac{y_1}{0.5}\right), \left(\frac{y_2}{0.3}\right)\right\}$ is $g_f - (i, j) - \text{pre} - \text{open set}$ in X but not $g_f - (i, j) - \alpha - \text{open set}$ in X.

Definition 4.6: A mapping $f: (X, t_1, t_2) \to (Y, g_1, g_2)$ is said to be pairwise $g_f - \beta - \text{continuous map}$ if inverse image of every $g_i - g_f - \text{open set}$ in Y is $g_f - (i, j) - \beta - \text{open set}$ in X

Remark 4.6: In a $g_f$ −bi-topological space$(X, t_1, t_2)$every pairwise $g_f - \text{continuous map}$ is pairwise $g_f - \beta - \text{continuous map}$ but not converse in general which is shown in Example 4.9

Example 4.10: In Example 4.5, the mapping $f: (X, t_1, t_2) \to (Y, g_1, g_2)$is pairwise $g_f - \beta - \text{continuous map}$ but not pairwise $g_f - \text{continuous}$ map the reason is same as in Example 4.5

Remark 4.7: In a $g_f$ −bi-topological space $(X, t_1, t_2)$ every pairwise $g_f - \text{semi} - \text{continuous map}$ is pairwise $g_f - \beta - \text{continuous map}$ but not converse in general which is shown in Example 4.10

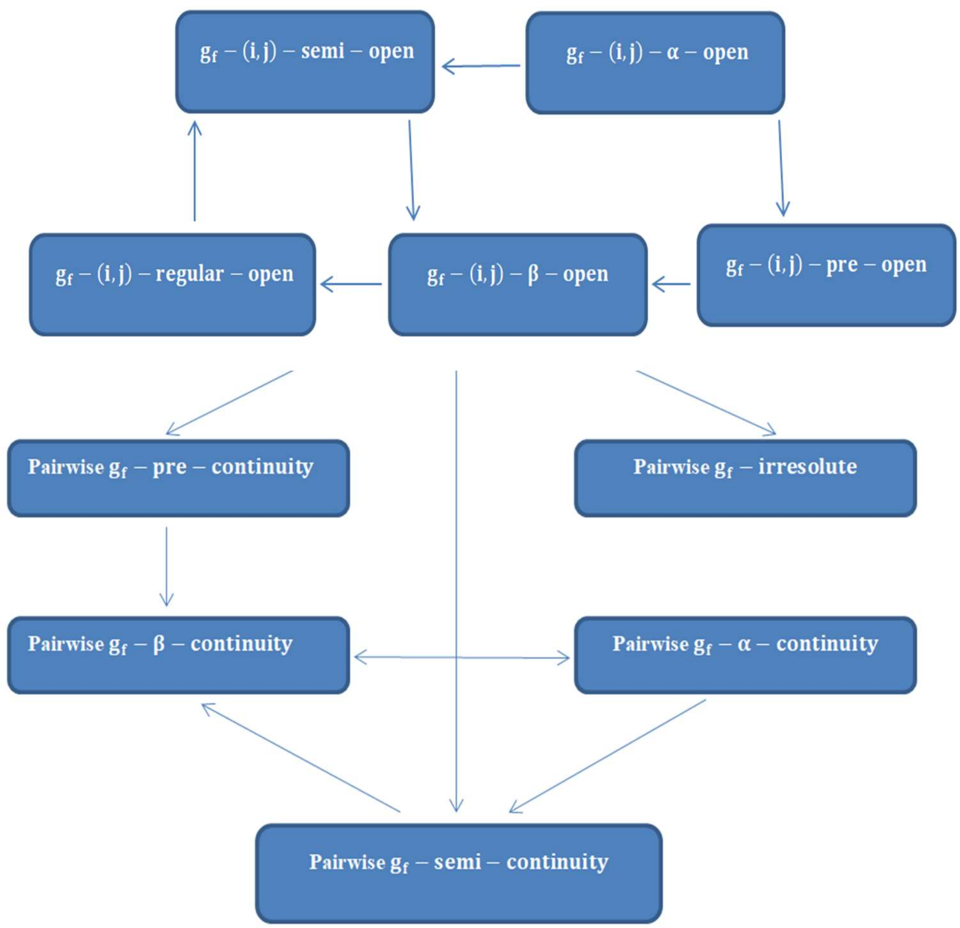
Example 4.11: In Example 4.8, the mapping $f: (X, t_1, t_2) \to (Y, g_1, g_2)$is pairwise $g_f - \text{semi} - \text{continuous map}$ but not pairwise $g_f - \text{pre} - \text{continuous map}$ because the set $S = \left\{\left(\frac{y_1}{0.5}\right), \left(\frac{y_2}{0.5}\right)\right\}$ is in X but not $g_f - (i, j) - \text{pre} - \text{open set}$ in X.

Remark 4.8: In a $g_f$ −bi-topological space $(X, t_1, t_2)$every pairwise $g_f - \text{pre} - \text{continuous map}$ is pairwise $g_f - \beta - \text{continuous map}$ but not converse in general which is shown in Example 4.11

Example 4.12: In Example 4.8, the mapping $f: (X, t_1, t_2) \to (Y, g_1, g_2)$is pairwise $g_f - \beta - \text{continuous map}$ but not pairwise $g_f - \text{pre} - \text{continuous map}$ because the set $S = \left\{\left(\frac{y_1}{0.5}\right), \left(\frac{y_2}{0.5}\right)\right\}$ is $g_f - (i, j) - \beta - \text{open set}$ in X but not $g_f - (i, j) - \text{pre} - \text{open set}$ in X.

## 5. Conclusion

In this paper we have studied a new concept of $g_f$ −bi-topological spaces in which many important results have been obtained and established the relationships with the help of some counter examples. Further we have studied a new concept of continuity in $g_f$ −bi-topological spaces in which many important results have been obtained. The following implications are the direct consequences of the definitions and various results that we discussed in this paper

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