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# AUM BLOCK DUPLICATION BY AN EDGE FOR TRIANGULAR SNAKE GRAPH AND ALTERNATE TRIANGULAR SNAKE GRAPH

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**Abstract**

*The concept of AUM Block Duplication is newly introduced Technique. In this paper AUM Block Duplication is applied to triangular snake graphs and Alternate triangular snake graphs. In particular a block in $T_2$ , $T_3$ , $T_4$ , $T_n(n \geq 5)$, $AT_2$ , $AT_3$ , $AT_4$ and $AT_n(n \geq 5)$ is duplicated by an edge. Suitable examples are given.*

**Keywords:** Duplication of a block, triangular snake graphs, alternate triangular snake graph.
**AMS classification:** 05C78.

## 1. Introduction:

Graph theory has become a vital tool in various fields including computer science, biology and social networks. Triangular snake graphs and alternate snake graphs are specific types of graphs which is applicable in network analysis and biological network, in particular protein – protein interaction network. Uma Maheswari et.al. first presented the block labelling technique in [3] [4] [5] [6] [7] [8] [9] [10] [11] and [12]. In [6] and [8], AUM Block colouring was examined. In [13], the authors developed AUM Block Sum Prime Distance Labelling for snake families of graphs. In [14] and [15], the authors presented the idea of block duplication for path graphs.

This paper examines the duplication of a block by an edge in triangular snake graphs and alternate triangular snake graphs.

## 2. Preliminaries:

**Definition 2.1[1]:** The graph $G$ is said to be separable if it has at least one cut-vertex. Otherwise, G is non-separable. A maximal non separable connected subgraph of graph $G$ is called a **block of graph $G$.**

**Definition 2.2[2]:** The triangular snake graph $T_n$ is obtained from the path $P_n$ by replacing each edge of the path by a triangle $C_3$

**Definition 2.3[2]:** An alternate triangular snake graph $A(T_n)$ is obtained from a path $P_n$ with vertices $u_1, u_2, \ldots, u_n$ by joining $u_i, u_{i+1}$ (alternatively) to a new vertex $v_i$

**Definition 2.4[14]:** Let $G$ be any Graph. **Neighbourhood of a block** B is the set of all blocks that have a common vertex with B and it is denoted by N(B).

**Definition 2.5[14]: AUM Block Duplication:** Let $G$ be any Graph. Duplication of a block $B_i$ by an edge e is the graph which is obtained by adding the new edge e = $uv$ to $G$ and joining the vertices $u$ and $v$ with the vertex common to $B_i$ and its neighbouring blocks $B_j$ .

The graph obtained after the duplication of a graph $G$ is called block duplicated graph and it is denoted by $D_G\ (B)$.

## 3. Duplication of a block by an edge

In this section we apply the block duplication by an edge to triangular snake graph and alternate triangular snake graph.

**PROPOSITION 3.1:**

Duplication of a block B in triangular snake graph $T_2$ is the duplicated graph $D_{T_2}(B)$(as in Figure 1)

**Proof:**

Let $u_1$,$u_2$, $u_3$ be the vertices and $B$ be the block of the graph $T_2$.

Let us duplicate the block $B$ by introducing a new edge $u_1'\ u_3'$. Duplication of a block $B$ is done by joining $u_1'$ and $u_3'$ with the vertices common to the block $B_1$ and its neighbours. But, we have only one block $B$, it has no neighbouring blocks, so there is no common vertex. Hence the duplicated graph will be a disconnected graph as in figure 1.

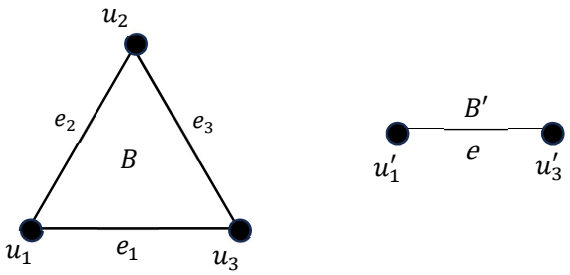


**Figure 1:** Duplicated graph $D_{T_2}(B)$

**PROPOSITION 3.2:**

Duplication of a block $B_i$ in triangular snake graph $T_3$ is the duplicated graph $D_{T_3}(B_i)$, $i = 1,2$ (as in Figure 2)

**Proof:**

Let $u_1$,$u_2$, $u_3$, $u_4$ , $u_5$ be the vertices and $B_1$ , $B_2$ be the blocks of the graph $T_3$.

**Case (i):** Duplication of the block $B_1$ :

Here $B_2$ is the neighboring block and $u_3$ is the common vertex to $B_1$ and its neighbor $B_2$.

Introduce the new edge $vw$ for duplication of a block $B_1$. Join the vertices $v$ and $w$ with the vertex $u_3$. Then the duplicated graph of $T_3$ is obtained as follows

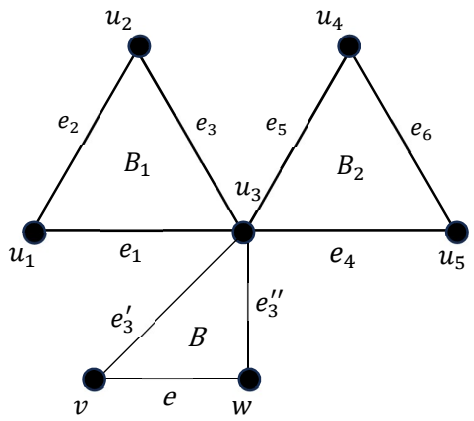


**Figure 2:** Duplicated graph $D_{T_3}(B_1)$

**Case (ii):** Duplication of the block $B_2$ :

Here $B_1$ is the neighboring block and $u_3$ is the common vertex to $B_2$ and its neighbor $B_1$.

Introduce the new edge $vw$ for duplication of the block $B_2$. Join the vertices $v$ and $w$ with the vertex $u_3$. Then the duplicated graph of $T_3$ is also obtained as in figure 2

**PROPOSITION 3.3:**

Duplication of a block $B_i$ in triangular snake graph $T_4$ is the duplicated graph $D_{T_4}(B_i)$, $1 \leq i \leq 3$ (as in Figure 3)

**Proof:**

Let $u_1, u_2, u_3, u_4, u_5, u_6, u_7$ be the vertices and $B_1, B_2, B_3$ be the blocks in $T_4$.

**Case (i):** Duplication of a block $B_1$ in $T_4$ :

Here $B_2$ is the neighboring block and $u_3$ is the common vertex to $B_1$ and its neighbor $B_2$.

Introduce the new edge $vw$ for duplication of a block $B_1$. Join the vertices $v$ and $w$ with the vertex $u_3$. Then the duplicated graph of $T_4$ is obtained as follows

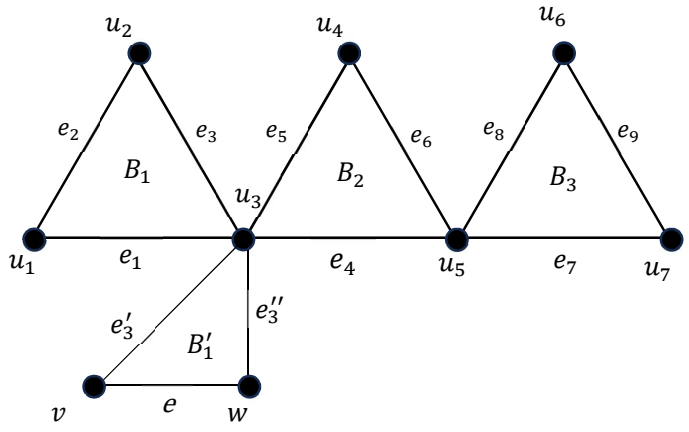


**Figure 3:** Duplicated graph $D_{T_4}(B_1)$

**Case (ii):** Duplication of a block $B_2$ in $T_4$ :

Here $B_1$ and $B_3$ are the neighboring blocks for $B_2$.

$\therefore u_3$ and $u_5$ are the vertices common to the neighboring blocks $B_1$ and $B_3$ of $B_2$ respectively.

Let us introduce the new edge $vw$ with the end vertices $v$ and $w$ for duplication of the block $B_2$. Join both the vertices $v$ and $w$ with $u_3$ and $u_5$.

Then the resulting graph is called the duplicated graph $D_{T_4}(B_2)$.

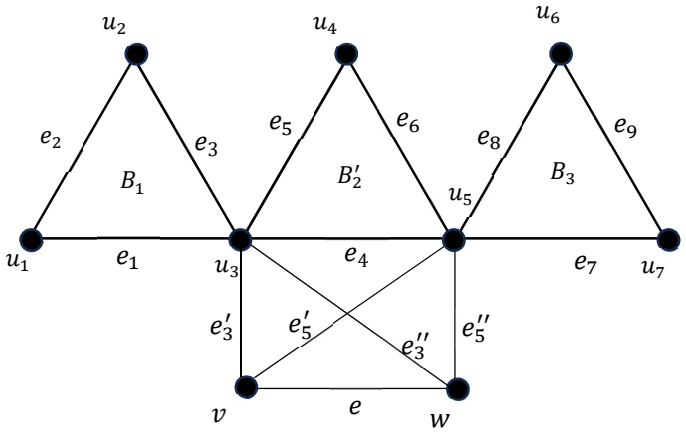


**Figure 4:** Duplicated graph $D_{T_4}(B_2)$

**Case (iii)**: Duplication of the block $B_3$ in $T_4$ :

Here $B_2$ is the only neighboring block of $B_3$ and $u_5$ is the common vertex for the block $B_3$ and its neighboring block $B_2$.

Let us consider the edge $vw$ with the end vertices $v$ and $w$ for duplicating the block $B_3$.

Join the vertices $v$ and $w$ the vertex $u_5$ for duplication of $B_3$. Then the duplicated graph of the graph $T_4$ is obtained as follows,

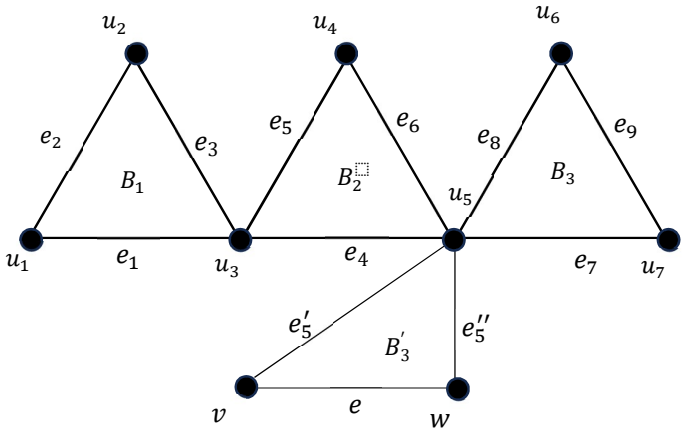


**Figure 5:** Duplicated graph $D_{T_4}(B_3)$

**PROPOSITION 3.4:**

Duplication of a block $B_i$ in triangular snake graph $T_n(n \geq 5)$ is the duplicated graph $D_{T_n}(B_i), 1 \leq i \leq n-1$ (as in Figure 6,7,8)

**Proof:**

Let $u_1, u_2, \ldots u_{2n-1}$ be the $2n-1$ vertices and $B_1, B_2, \ldots B_{n-1}$ be the $n-1$ blocks in $T_n$.

**Case (i):** Duplication of the block $B_1$ in $T_n$ .

Here $B_2$ is the neighbouring block and $u_3$ is the common vertex to $B_1$ and its neighbor $B_2$. Introduce the new edge $vw$ for duplication of a block $B_1$. Join the vertices $v$ and $w$ with the vertex $u_3$.

Then the duplicated graph of $T_n$ is obtained as follows

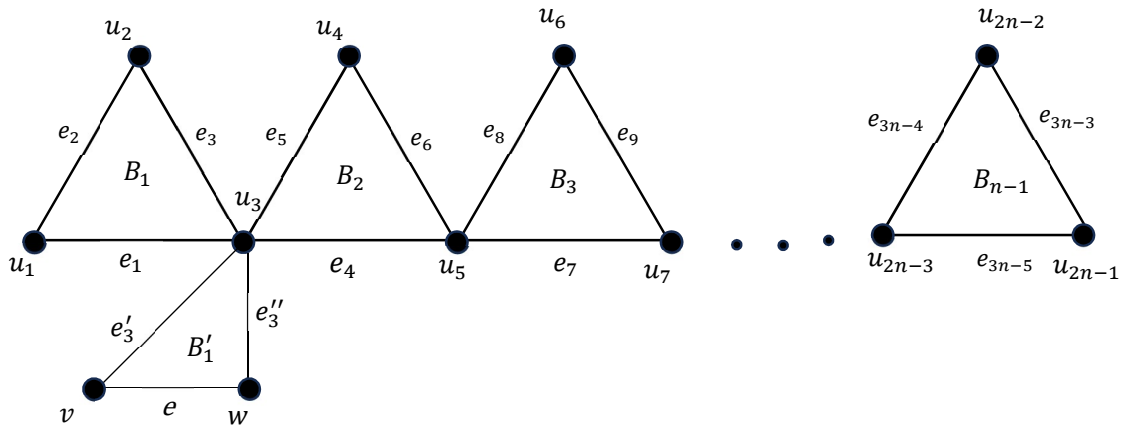


**Figure 6**: Duplicated graph $D_{T_n}(B_1)$

**Case (ii):** Duplication of the block $B_i$ , $(2 \leq i \leq n-2)$ in $T_n$.

Here $B_{i-1}$, $B_{i+1}$ are the neighboring blocks for $B_i$.

$\therefore u_{2i-1}$ , $u_{2i+1}$ are the vertices common to the neighboring blocks $B_{i-1}$ and $B_{i+1}$ of $B_i$ respectively.

Let us introduce the new edge $vw$ with the end vertices $v$ and $w$ for duplicating the block $B_i$. Joint the vertex $v$ with $u_{2i-1}$ and $u_{2i+1}$ and $w$ with $u_{2i-1}$ and $u_{2i+1}$ .

Then the duplicated graph is obtained as follows,

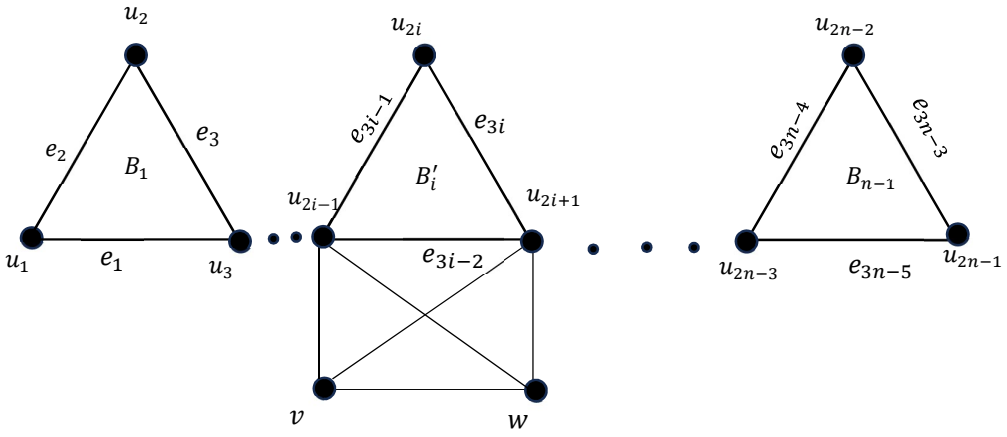


**Figure 7:** Duplicated graph $D_{T_n}(B_i)$

**Case (iii):** Duplication of the block $B_{n-1}$ in $T_n$.

Here $B_{n-2}$ is the only neighbouring block of $B_{n-1}$ and $u_{2n-3}$ is the common vertex for the block $B_{n-1}$ and its neighboring block $B_{n-2}$.

Let us consider the edge $vw$ with the end vertices $v$ and $w$ for duplicating the block $B_{n-1}$.

Join the vertices $v$ and $w$ with the vertex $u_{2n-3}$.

Then the duplicated graph of $T_n$ is obtained as follows,

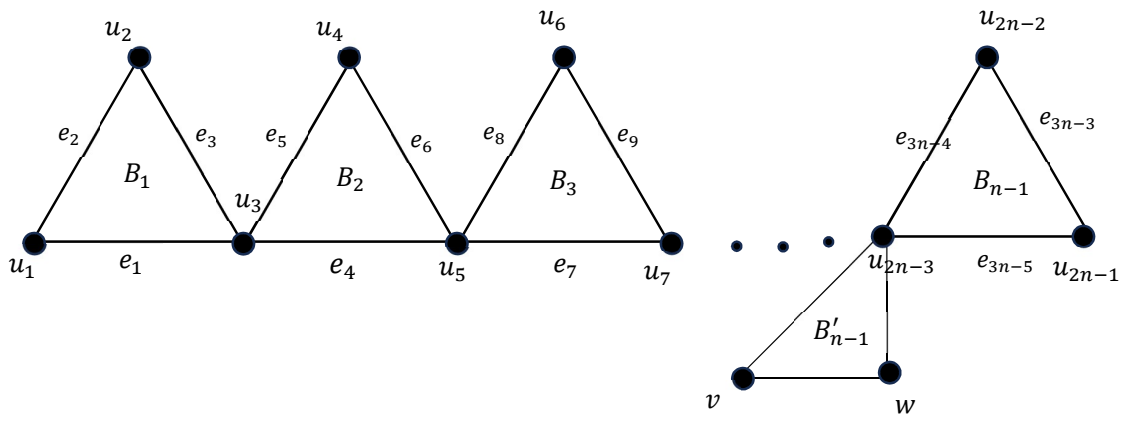


**Figure 8:** Duplicated graph $D_{T_n}(B_{n-1})$

**PROPOSITION 3.5:**

Duplication of a block B in alternate Triangular snake graph $AT_2$ is the duplicated graph $D_{AT_2}(B)$(as in Figure 9)

**Proof:**

Alternate Triangular snake graph $AT_2$ is same as triangular snake graph $T_2$. Hence the duplication of $AT_2$is same as duplication of $T_2$. (Refer Proposition 3.1)

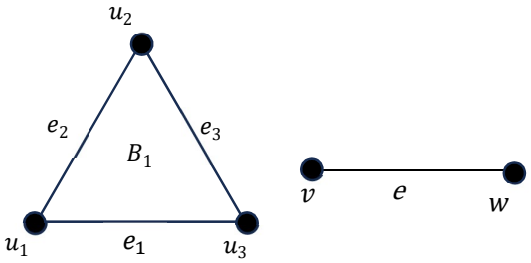


**Figure 9:** Duplicated graph $D_{AT_2}(B)$

**PROPOSITION 3.6:**

Duplication of a block $B_i$ in alternate Triangular snake graph $AT_3$ is the duplicated graph $D_{AT_3}(B_i)$, $i = 1,2$ (as in Figure 10)

**Proof:**

Let $u_1$,$u_2$, $u_3$, $u_4$ be the vertices and $B_1$ , $B_2$ be the blocks of the graph $AT_3$.

**Case (i):** Duplication of the block $B_1$ :

Here $B_2$ is the neighboring block and $u_3$ is the common vertex to $B_1$ and its neighbor $B_2$.

Introduce the new edge $vw$ for duplication of a block $B_1$. Join the vertices $v$ and $w$ with the vertex $u_3$. Then the duplicated graph of $AT_3$ is obtained as follows

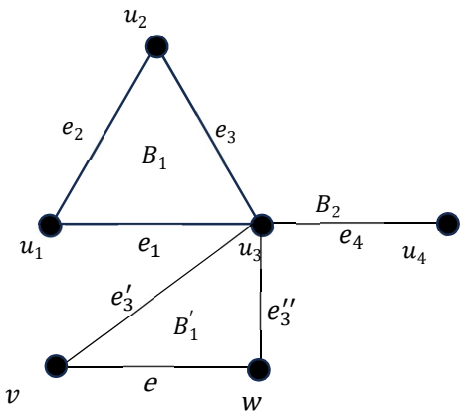


**Figure 10:** Duplicated graph $D_{AT_3}(B_i)$

**Case (ii):** Duplication of the block $B_2$ :
Here $B_1$ is the neighboring block and $u_3$ is the common vertex to $B_2$ and its neighbor $B_1$.
Introduce the new edge $vw$ for duplication of the block $B_2$. Join the vertices $v$ and $w$ with the vertex $u_3$. Then the duplicated graph of $AT_3$ is also obtained as in figure 10.

**PROPOSITION 3.7:**

Duplication of a block $B_i$ in alternate triangular snake graph $AT_4$ is the duplicated graph $D_{AT_4}(B_i)$, $1 \leq i \leq 3$ (as in Figure 11, 12, 13)

**Proof:** Let $u_1, u_2, u_3, u_4, u_5, u_6$ be the vertices and $B_1, B_2, B_3$ be the blocks in $AT_4$.
**Case (i):** Duplication of a block $B_1$ in $AT_4$ :
Here $B_2$ is the neighboring block and $u_3$ is the common vertex to $B_1$ and its neighbor $B_2$.
Introduce the new edge $vw$ for duplication of a block $B_1$. Join the vertices $v$ and $w$ with the vertex $u_3$. Then the duplicated graph of $AT_4$ is obtained as follows

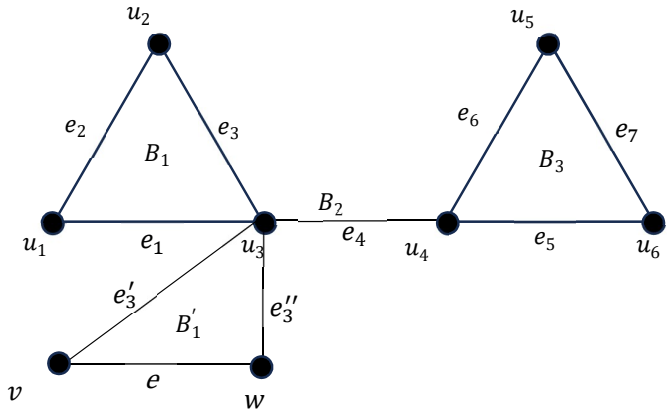


**Figure 11:** Duplicated graph $D_{AT_4}(B_1)$
**Case (ii):** Duplication of a block $B_2$ in $AT_4$ :
Here $B_1$ and $B_3$ are the neighboring blocks for $B_2$.
$\therefore$ $u_3$ and $u_4$ are the vertices common to the neighboring blocks $B_1$ and $B_3$ of $B_2$ respectively.

Let us introduce the new edge $vw$ with the end vertices $v$ and $w$ for duplication of the block $B_2$. Join both the vertices $v$ and $w$ with $u_3$ and $u_4$.
Then the resulting graph is called the duplicated graph $D_{AT_4}(B_2)$.

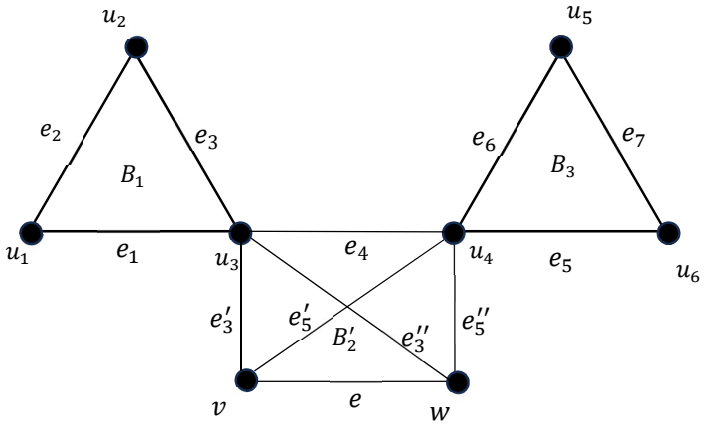


**Figure 12:** Duplicated graph $D_{AT_4}(B_2)$

**Case (iii)**: Duplication of the block $B_3$ in $AT_4$ :
Here $B_2$ is the only neighboring block of $B_3$ and $u_4$ is the common vertex for the block $B_3$ and its neighboring block $B_2$.
Let us consider the edge $vw$ with the end vertices $v$ and $w$ for duplicating the block $B_3$.
Join the vertices $v$ and $w$ the vertex $u_4$ for duplication of $B_3$. Then the duplicated graph of the graph $AT_4$ is obtained as follows,

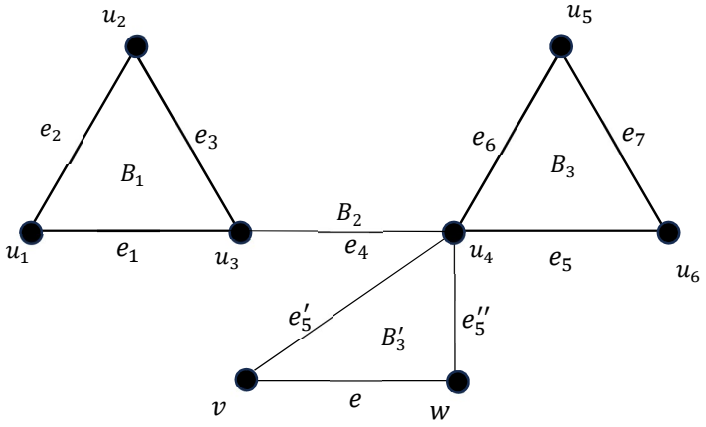


**Figure 13:** Duplicated graph $D_{AT_4}(B_3)$

**PROPOSITION 3.8:**
Duplication of a block $B_i$ in alternate triangular snake graph $AT_n(n \geq 5)$ is the duplicated graph $D_{AT_n}(B_i)$ , $1 \leq i \leq n-1$ (as in Figure 14,15,16)

**Proof:**

Let $G = AT_n$ , $n \geq 5$ be an alternate triangular snake graph. Let $V(G) = \{ u_1, u_2, u_3, \ldots, u_{\lfloor \frac{3n}{2} \rfloor} \}$. $E(G) = \{ e_1 , e_2 , \ldots , e_{2n-1} \}$, if n is even. $E(G) = \{ e_1 , e_2 , \ldots , e_{2n-2} \}$, if $n$ is odd and $B(G) = \{B_1 , B_2 , \ldots , B_{n-1} \}$ be the vertex set, edge set and block set of $G$ respectively.

**Case (i):** Duplication of the block $B_1$ in $AT_n$ .
Here $B_2$ is the neighbouring block and $u_3$ is the common vertex to $B_1$ and its neighbor $B_2$. Introduce the new edge $vw$ for duplication of a block $B_1$. Join the vertices $v$ and $w$ with the vertex $u_3$.
Then the duplicated graph of $AT_n$ is obtained as follows

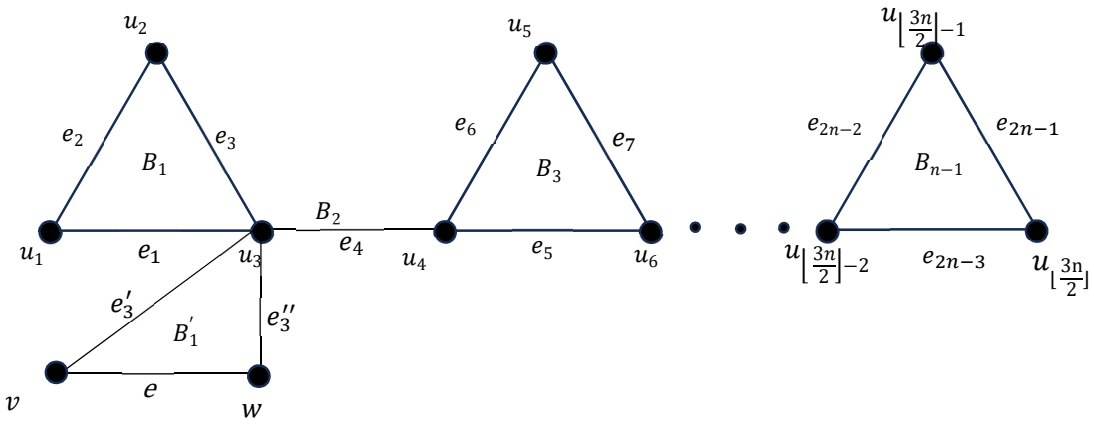


**Figure 14:** the duplicated graph $D_{AT_n}(B_1)$

**Case (ii):** Duplication of the block $B_i$ , $(2 \leq i \leq n-2)$ in $AT_n$.
Here $B_{i-1}$, $B_{i+1}$ are the neighboring blocks for $B_i$.
$\therefore u_{3i-2}$ , $u_{3i}$ are the vertices common to the neighboring blocks $B_{i-1}$ and $B_{i+1}$ of $B_i$ respectively.
Let us introduce the new edge $vw$ with the end vertices $v$ and $w$ for duplicating the block $B_i$. Join the vertex $v$ with $u_{3i-2}$ and $u_{3i}$ and join $w$ with $u_{3i-2}$ and $u_{3i}$.
Then the duplicated graph is obtained as follows,

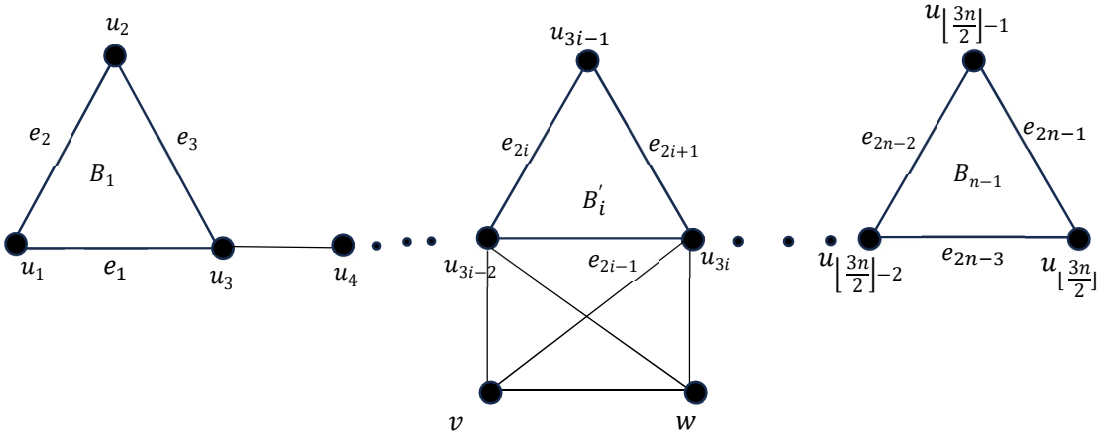


**Figure 15:** the duplicated graph $D_{AT_n}(B_i)$

**Case (iii):** Duplication of the block $B_{n-1}$ in $AT_n$.

Here $B_{n-2}$ is the only neighbouring block of $B_{n-1}$ and $u_{\lfloor \frac{3n}{2} \rfloor - 2}$ is the common vertex for the block $B_{n-1}$ and its neighboring block $B_{n-2}$.

Let us consider the edge $vw$ with the end vertices $v$ and $w$ for duplicating the block $B_{n-1}$.

Join the vertices $v$ and $w$ with the vertex $u_{\lfloor \frac{3n}{2} \rfloor - 2}$.

Then the duplicated graph of $AT_n$ is obtained as follows,

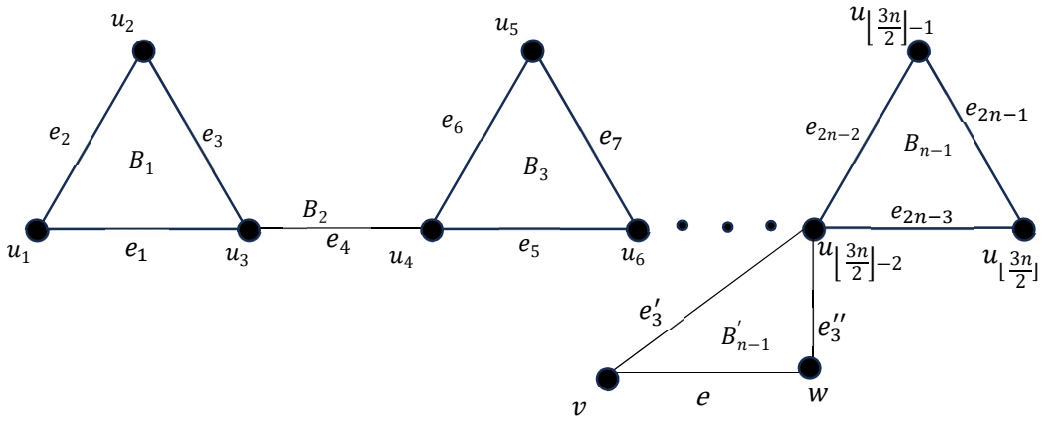


**Figure 16:** the duplicated graph $D_{AT_n}(B_{n-1})$

**Conclusion:**

This paper has investigated the duplication of a block by an edge in triangular snake graphs and alternate triangular snake graphs. Also, the suitable examples are given wherever necessary. There is a scope for extending the results to other families of graphs with applications in social network, biological network and so on.

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